Bruneau, Laurent; Dereziński, Jan; Georgescu, Vladimir
Homogeneous Schrödinger operators on half-line. (English) Zbl 1226.47049
Ann. Henri Poincaré 12, No. 3, 547-590 (2011).

This article is devoted to a thorough study of the differential expression $L_m := -\partial_x^2 + (m^2 - 1/4)x^{-2}$ on $C_c^\infty(0,\infty)$, and of the operators it induces in the complex space $L^2(0,\infty)$. These operators appear in the decomposition of the Aharonov-Bohm Hamiltonian. They are also of interest due to their role in the theory of special functions.

The results state essentially that there is a unique holomorphic family $\{H_m\}_{\mathrm{Re}\ m>-1}$ of closed operators in L^2 such that H_m coincides with the closure of L_m for $m \geq 1$. The operators H_m are homogeneous of degree -2 with respect to the group of dilations in L^2. The spectrum and essential spectrum of H_m is $[0,\infty)$, independently of m. The numerical range of H_m is calculated explicitly. If $\mathrm{Re}\ m > -1$, $\mathrm{Re}\ k > -1$ and $\lambda \in \mathbb{C} \setminus [0,\infty)$, then $(H_m - \lambda)^{-1} - (H_k - \lambda)^{-1}$ is a compact operator. There are also results on the scattering theory for H_m, e.g., an explicit expression for the wave operators.

The proofs rest on an abstract study of operators that are homogeneous with respect to a strongly continuous group of unitary operators in a Hilbert space. Moreover, explicit formulas and estimates involving Bessel functions and the Hankel transform are employed.

Reviewer: Nils Ackermann (Mexico City)

MSC:

47E05	General theory of ordinary differential operators
34L40	Particular ordinary differential operators (Dirac, one-dimensional Schrödinger, etc.)
34L25	Scattering theory, inverse scattering involving ordinary differential operators
47A55	Perturbation theory of linear operators
47A40	Scattering theory of linear operators
47A20	Dilations, extensions, compressions of linear operators
47A12	Numerical range, numerical radius
81Q12	Nonselfadjoint operator theory in quantum theory including creation and destruction operators
81Q05	Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics

Keywords:

homogeneous operator; holomorphic family of closed operators; Aharonov-Bohm Hamiltonian; Hankel transform; group of dilations

Full Text: DOI arXiv

References:

[1] Everitt, W. N.; Kalf, H., The Bessel differential equation and the Hankel transform, J. Comput. Appl. Math., 208, 3-19 (2007) · Zbl 1144.34013 · doi:10.1016/j.cam.2006.10.029

[2] Kato, T.: Perturbation theory for linear operators · Zbl 0148.12601

[3] Kellendonk, J., Richard, S.: Weber-Schafheitlin type integrals with exponent 1. Int. Transforms Special Funct. 20 (2009) · Zbl 1163.33313

[4] Le Yaouanc, A.; Oliver, L.; Raynal, J.-C., The Hamiltonian (p^2 + m^2)^1/2 − α/r near the critical value α_c = 2/π, J. Math. Phys., 38, 3997-4012 (1997) · Zbl 0883.47084 · doi:10.1063/1.532106

[5] Pankrashkin, K., Richard, S.: Spectral and scattering theory for the Aharonov-Bohm operators (preprint) (2009). arXiv:0911.4715 · Zbl 1230.81054

[6] Stone, M.H.: Linear transformations in Hilbert space. Am. Math. Soc. (1932) · Zbl 58.0420.02

[7] Watson, G. N., A Treatise on the Theory of Bessel Functions (1995), Cambridge: Cambridge University Press, Cambridge · Zbl 0849.33001

[8] Zhu, K.: Operator theory in functions spaces, 2nd edn. Mathematical Surveys and Monographs, vol. 138, ISSN 0076-5376 (2007)

