

**Goldberg, Michael**

**A dispersive bound for three-dimensional Schrödinger operators with zero energy eigenvalues.** (English) Zbl 1223.35265

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Consider  $V \in L^p(\mathbb{R}^3) \cap L^q(\mathbb{R}^3)$  for exponents  $p < \frac{3}{2} < q$ . Here all function spaces are complex and over  $\mathbb{R}^3$ . Denote by  $H := -\Delta + V$  the corresponding Schrödinger operator, which may not be symmetric. A resonance  $\Psi$  of  $H$  is a distributional solution of  $H\Psi = \lambda^2\Psi$ , for some  $\lambda \in \mathbb{R}$ , such that  $\Psi \in L^3_{\text{weak}} \setminus L^2$ . Denote by  $X_1$  the set of  $\Psi \in L^2$  that are solutions of  $H\Psi = 0$ , that is, the zero energy eigenfunctions. Define inductively, if  $X_k \subseteq L^1$ , the space

$$X_{k+1} := \{\Psi \in L^3_{\text{weak}} \mid H\Psi \in X_k\}.$$

The main result is a bound on the  $L^\infty$ -norm of the time evolution of initial values  $f \in L^1$  away from the generalized eigenspaces of  $H$ , in terms of the  $L^1$ -norm of  $f$ . Suppose that  $H$  has no resonances, that  $X_k \subseteq L^1$  for each  $k \in \mathbb{N}$ , and that  $\bigcup_{k \in \mathbb{N}} X_k$  is finite dimensional. Setting  $P$  to the sum of all spectral projections to generalized eigenspaces of eigenvalues of  $H$ , there is  $C > 0$  such that

$$\|e^{-itH}(I - P)f\|_\infty \leq C|t|^{-3/2}\|f\|_1$$

for all  $f \in L^1$  and

$$\|e^{-itH}(I - P)f\|_2 \leq C\|f\|_2$$

for all  $f \in L^2$ .

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**MSC:**

- 35Q41 Time-dependent Schrödinger equations and Dirac equations
- 81U30 Dispersion theory, dispersion relations arising in quantum theory
- 35J10 Schrödinger operator, Schrödinger equation
- 47D08 Schrödinger and Feynman-Kac semigroups

Cited in 14 Documents

**Keywords:**

dispersive bound; resonance; generalized eigenvalue; spectral projection

**Full Text:** [DOI](#) [arXiv](#)

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