Goldberg, Michael

A dispersive bound for three-dimensional Schrödinger operators with zero energy eigenvalues. (English) Zbl 1223.35265

Commun. Partial Differ. Equations 35, No. 9, 1610-1634 (2010).

Consider $V \in L^p(\mathbb{R}^3) \cap L^q(\mathbb{R}^3)$ for exponents $p < \frac{3}{2} < q$. Here all function spaces are complex and over $\mathbb{R}^3$. Denote by $H := -\Delta + V$ the corresponding Schrödinger operator, which may not be symmetric. A resonance Ψ of H is a distributional solution of $H\Psi = \lambda^2\Psi$, for some $\lambda \in \mathbb{R}$, such that $\Psi \in L^3_{\text{weak}} \setminus L^2$. Denote by X_1 the set of $\Psi \in L^2$ that are solutions of $H\Psi = 0$, that is, the zero energy eigenfunctions. Define inductively, if $X_k \subseteq L^1$, the space

$$X_{k+1} := \{\Psi \in L^3_{\text{weak}} \mid H\Psi \in X_k\}.$$

The main result is a bound on the L^∞-norm of the time evolution of initial values $f \in L^1$ away from the generalized eigenspaces of H, in terms of the L^1-norm of f. Suppose that H has no resonances, that $X_k \subseteq L^1$ for each $k \in \mathbb{N}$, and that $\bigcup_{k\in\mathbb{N}} X_k$ is finite dimensional. Setting P to the sum of all spectral projections to generalized eigenspaces of eigenvalues of H, there is $C > 0$ such that

$$\|e^{-itH}(I-P)f\|_\infty \le C|t|^{-3/2}\|f\|_1$$

for all $f \in L^1$ and

$$\|e^{-itH}(I-P)f\|_2 \le C\|f\|_2$$

for all $f \in L^2$.

Reviewer: Nils Ackermann (Mexico City)

MSC:

35Q41 Time-dependent Schrödinger equations and Dirac equations
81U30 Dispersion theory, dispersion relations arising in quantum theory
35J10 Schrödinger operator, Schrödinger equation
47D08 Schrödinger and Feynman-Kac semigroups

Keywords:

dispersive bound; resonance; generalized eigenvalue; spectral projection

Full Text: DOI arXiv

References:

[1] Agmon S., Ann. Sc. Norm. Super. Pisa. Cl. Sci. (4) 2 pp 151– (1975)
[2] Arveson W., A Short Course on Spectral Theory (2002)
[3] DOI: 10.1007/s00220-008-0427-3 · Zbl 1148.35082 · doi:10.1007/s00220-008-0427-3
[4] DOI: 10.1002/cpa.1018 · Zbl 1031.35129 · doi:10.1002/cpa.1018
[5] Erdogan M.B., Dyn. Partial Differ. Equ. 1 pp 359– (2004)
[6] DOI: 10.1007/BF02789446 · Zbl 1146.35324 · doi:10.1007/BF02789446
[7] Goldberg M., Geom. Funct. Anal. 16 pp 517– (2006)
[8] DOI: 10.1016/j.jfa.2008.11.005 · Zbl 1161.35004 · doi:10.1016/j.jfa.2008.11.005
[9] DOI: 10.1007/s00220-004-1140-5 · Zbl 1086.81077 · doi:10.1007/s00220-004-1140-5
[10] DOI: 10.1007/s00039-003-0439-2 · Zbl 1055.35098 · doi:10.1007/s00039-003-0439-2
[11] DOI: 10.1215/S0012-7094-79-04631-3 · Zbl 0448.35080 · doi:10.1215/S0012-7094-79-04631-3
[12] DOI: 10.1002/cpa.3160440504 · Zbl 0743.35008 · doi:10.1002/cpa.3160440504
[13] DOI: 10.1353/ajm.1998.0039 · Zbl 0922.35028 · doi:10.1353/ajm.1998.0039

[14] DOI: 10.1215/S0012-7094-87-05518-9 · Zbl 0644.35012 · doi:10.1215/S0012-7094-87-05518-9

[15] Reed M., Methods of Modern Mathematical Physics. II. Fourier Analysis, Self Adjointness (1975) · Zbl 0308.47002

[16] Reed M., Methods of Modern Mathematical Physics. I. Functional Analysis (1980) · Zbl 0459.46001

[17] DOI: 10.1007/s00222-003-0325-4 · Zbl 1063.35035 · doi:10.1007/s00222-003-0325-4

[18] DOI: 10.1002/cpa.20066 · Zbl 1130.81053 · doi:10.1002/cpa.20066

[19] DOI: 10.1142/S0129055X04002175 · Zbl 1111.81313 · doi:10.1142/S0129055X04002175

[20] DOI: 10.2969/jmsj/04730551 · Zbl 0837.35039 · doi:10.2969/jmsj/04730551

[21] DOI: 10.1007/s00220-005-1375-9 · Zbl 1079.81021 · doi:10.1007/s00220-005-1375-9