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Structural properties of the set of global solutions of the nonlinear heat equation. (English)

Zbl 1222.35108

Aiki, T. (ed.) et al., Current advances in nonlinear analysis and related topics. Collected papers of the conference on nonlinear evolution equations and related topics, Tokyo, Japan, October 10–12, 2009, the 4th Polish-Japanese days on current advances in applied nonlinear analysis and mathematical modelling issues, Warsaw, Poland, May 18–21, 2009 and the RIMS conference on nonlinear evolution equations and mathematical modeling, Kyoto, Japan, October 20–24, 2009. Tokyo: Gakkōtoshō (ISBN 978-4-7625-0457-0/hbk). GAKUTO International Series. Mathematical Sciences and Applications 32, 13-23 (2010).

Consider the semilinear parabolic problem

$$\begin{cases} u_t - \Delta u = f(u) & \text{on } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ u(0) = u_0, \end{cases} \quad (1)$$

where Ω is a bounded smooth domain in \mathbb{R}^N . The authors consider geometrical and topological properties of the set \mathcal{G} of values $u_0 \in C_0(\bar{\Omega})$ initiating globally defined solutions and of the set \mathcal{B} of values $u_0 \in C_0(\bar{\Omega})$ initiating solutions that blow up in finite time. Here $C_0(\bar{\Omega})$ denotes the space of continuous functions on $\bar{\Omega}$ that vanish on $\partial\Omega$. Note that $\mathcal{G} \cup \mathcal{B} = C_0(\bar{\Omega})$.

Suppose that $f \in C^1(\mathbb{R}, \mathbb{R})$ and that there is $\varepsilon > 0$ such that $s^2 f'(s) \geq (1 + \varepsilon) s f(s)$ for all $s \in \mathbb{R}$. The main results are: If in addition $s f(s)$ is positive for large values of $|s|$ and if f has at most subcritical growth, then \mathcal{G} is not convex. If $f(s) = |s|^\alpha s$ with $\alpha > 0$, if f has at most subcritical growth, and if Ω is a ball, then the set \mathcal{G}_{rad} of radially symmetric functions initiating global solutions of (1) is connected. And, last but not least, the set \mathcal{B} is path connected in general.

There is also a section about open problems.

For the entire collection see [Zbl 1192.35004].

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MSC:

- 35K58 Semilinear parabolic equations
- 35B44 Blow-up in context of PDEs
- 35B40 Asymptotic behavior of solutions to PDEs

Cited in 3 Documents

Keywords:

convexity; connectedness; blow-up set; global existence; subcritical growth