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Numerical wave propagation on the hexagonal C-grid. (English) Zbl 1220.76018

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Summary: Inertio-gravity mode and Rossby mode dispersion properties are examined for discretizations of the linearized rotating shallow water equations on a regular hexagonal C-grid in planar geometry. It is shown that spurious non-zero Rossby mode frequencies found by previous authors in the f-plane case can be avoided by an appropriate discretization of the Coriolis terms. Three generalizations of this discretization that conserve energy even for non-constant Coriolis parameter are presented. A quasi-geostrophic β-plane analysis is carried out to investigate the Rossby mode dispersion properties of these three schemes. The Rossby mode dispersion relation is found to have two branches. The primary branch modes are good approximations, in terms of both structure and frequency, to corresponding modes of the continuous governing equations, and offer some improvements over a quadrilateral C-grid scheme. The secondary branch modes have vorticity structures approximating those of small-scale modes of the continuous governing equations, suggesting that the hexagonal C-grid might have an advantage in terms of resolving extra Rossby modes; however, the frequencies of the secondary branch Rossby modes are much smaller than those of the corresponding continuous modes, so this potential advantage is not fully realized.

MSC:

76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction

65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

76U05 General theory of rotating fluids

86A05 Hydrology, hydrography, oceanography

Keywords:

staggered grid; Rossby mode; inertio-gravity mode; shallow-water equations; dispersion relation

Software:

NICAM

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