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The scattering matrix and associated formulas in Hamiltonian mechanics. (English)

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Authors' abstract: "We prove two new identities in scattering theory in Hamiltonian mechanics and discuss the analogy between these identities and their counterparts in quantum scattering theory. These identities involve the Poincaré scattering map, which is analogous to the scattering matrix. The first of our identities states that the Calabi invariant of the Poincaré scattering map can be expressed as the regularised phase space volume. This is analogous to the Birman-Krein formula. The second identity relates the Poincaré scattering map to the total time delay and is analogous to the Eisenbud-Wigner formula."

Reviewer: [Yuri E. Gliklikh \(Voronezh\)](#)

MSC:

- 37J05 Relations of dynamical systems with symplectic geometry and topology (MSC2010)
- 37J10 Symplectic mappings, fixed points (dynamical systems) (MSC2010)
- 34L25 Scattering theory, inverse scattering involving ordinary differential operators
- 35P25 Scattering theory for PDEs
- 81U20 *S*-matrix theory, etc. in quantum theory

Cited in **3** Documents

Keywords:

[Hamiltonian mechanics](#); [scattering matrix](#); [Poincaré scattering map](#); [classical and quantum scattering theory](#)

Full Text: [DOI](#) [arXiv](#)

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