Hofer, Helmut; Wysocki, Krzysztof; Zehnder, Eduard
A general Fredholm theory. II: Implicit function theorems. (English) Zbl 1217.58005
Geom. Funct. Anal. 19, No. 1, 206-293 (2009).

The paper under review is a sequel of Part I [J. Eur. Math. Soc. (JEMS) 9, No. 4, 841–876 (2007; Zbl 1149.53053)] where the authors aimed to construct a new theory of generalized differential geometry and polyfolds in infinite dimensional spaces such that the theory can be applied to the analysis for symplectic/instanton Floer theory, the Gromov-Witten theory and the symplectic/contact field theory from a unified approach. The main results in this paper develop the analysis for a generalized Fredholm theory for sections of strong M-polyfold bundles.

In Section 2, the authors introduce sc^0-contraction germs $f: E \to F$ for sc-Banach spaces E and F such that (1) f is a germ on each level m with respect to an sc-open neighborhood $\cdots \subset U_m \subset \cdots U_1 \subset U_0$ and $F_m \subset f(U_m)$, and (2) for every level m and $\varepsilon \in (0,1)$,

$$\|f(v,u) - f(v,u') - (u-u')\|_m \leq \varepsilon \|u-u'\|_m$$

for all (v,u) and (v,u') close to $(0,0)$.

The main result in Section 2 is Theorem 2.6 (a germ-implicit function theorem) for an sc^0-contraction germ with the same regularity. The crucial notion of polyfold Fredholm section is defined in Section 3. The polyfold Fredholm section is regularized with respect to bootstrapping in terms of sc^+-sections, and has a fillable representative under the strong equivalence relation. Section 4 gives the special property of the sc-structure and sc-section to have an infinitesimal smooth implicit function theorem for contraction germs. The authors point out that the important difference on the existence of a good parametrization requires much stronger hypotheses than the traditional Fredholm theorem. A local structure of the solution set of Fredholm sections of fillable strong M-polyfold bundles is given by Theorem 4.6 in the case of no boundary, and by Theorem 4.18 in the boundary case.

In Section 5, the Fredholm theory in M-polyfold bundles is developed:

(i) mixed convergence is invariant under M-polyfold chart transformations and plays an intrinsic notion for M-polyfold bundles;

(ii) an auxiliary norm is introduced in order to estimate the size of perturbations and the existence of an auxiliary norm for fillable strong M-polyfold bundles with reflexive fibres is established;

(iii) local compactness is proven in Theorem 5.9;

(iv) proper Fredholm sections and m-level properness are given in Theorem 5.11 and Theorem 5.12;

(v) the solution set is a smooth compact manifold for a proper and transversal Fredholm section without boundary by Theorem 5.14. The possible "bubbling" bad position of the solution set to the boundary may occur when the ambient M-polyfold has a boundary. The good position is defined in Definition 5.15, and Theorem 5.14 can be extended to the boundary case if the proper Fredholm section is in good position to the corner structure (see Theorem 5.16 and Theorem 5.18);

(vi) perturbation s of a Fredholm section f and the generation for a Fredholm section $f+s$ are obtained for fillable strong M-polyfold bundles and the sc-structure from separable sc-Hilbert spaces. This separable sc-Hilbert space admits sc-smooth partitions of unity on M-polyfolds as is shown in the first part of the paper. Hence, $(f+s)^{-1}(0)$ is a smooth compact manifold with boundary with corners;

(vii) the authors introduce some invariants of oriented proper Fredholm sections.

The paper carries many technical issues of generalized differential geometry. The glossary in Section 7 is useful to recall many basic notions. It would be more friendly to learn these notions with examples in symplectic/instanton Floer theory, the Gromov-Witten theory. The paper requires high energy to read through and digest, and we refer for details to the first part.

Reviewer: Weiping Li (Stillwater)

MSC:

58B15	Fredholm structures on infinite-dimensional manifolds
53D45	Gromov-Witten invariants, quantum cohomology, Frobenius manifolds
53D40	Symplectic aspects of Floer homology and cohomology
58C15	Implicit function theorems; global Newton methods on manifolds
47J07	Abstract inverse mapping and implicit function theorems involving nonlinear operators
53D42	Symplectic field theory; contact homology

Keywords:

sc-Banach space; *sc*-smoothness; M-polyfolds; Fredholm sections of M-polyfold bundles; implicit function theorem

Full Text: DOI