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Liouville-type theorems and asymptotic behavior of nodal radial solutions of semilinear heat equations. (English) Zbl 1215.35041

J. Eur. Math. Soc. (JEMS) 13, No. 1, 219-247 (2011).

Motivated by nonexistence results for positive solutions and for certain classes of sign-changing solutions of the elliptic problem $-\Delta u = |u|^{p-1}u$ posed on $\mathbb{R}^N$, the authors are concerned with Liouville-type theorems for classical radial (in x) entire solutions $u = u(x,t)$ of the parabolic problem

$$u_t - \Delta u = |u|^{p-1}u \qquad x \in \mathbb{R}^N,\ t \in \mathbb{R}, \tag{1}$$

where always $p > 1$.

Let p_S denote the critical Sobolev exponent $(N+2)/(N-2)$ if $N \geq 3$ and set $p_S := \infty$ if $N = 1,2$. It has been known before that if u is a nonnegative radial solution of (1) and if $p < p_S$, then $u \equiv 0$. In the case $N = 1$ no symmetry condition was needed at all. These theorems are extended to the case of sign changing functions as follows: If $p < p_S$ and if u is a classical x-radial solution of (1) such that the number of sign changes (aka, the zero number) of $u(\cdot,t)$ remains bounded for $t \in \mathbb{R}$, then $u \equiv 0$. Again, the symmetry condition in x is not needed if $N = 1$.

As an application of these results the authors prove a priori estimates for the decay and blow up rates of radial solutions of general semilinear parabolic problems on radial domains, where the constants in the estimates depend only on the data of the problem and an upper bound for the zero number along the orbit. The nonlinearity in these problems must behave asymptotically like a power function in u. To give one example for the utility of these estimates, existence results for periodic radial solutions with an arbitrarily prescribed zero number for a time-periodic semilinear parabolic equation on a ball are given.

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MSC:

35B53	Liouville theorems and Phragmén-Lindelöf theorems in context of PDEs
35B08	Entire solutions to PDEs
35B45	A priori estimates in context of PDEs
35K91	Semilinear parabolic equations with Laplacian, bi-Laplacian or poly-Laplacian
35K58	Semilinear parabolic equations
35B10	Periodic solutions to PDEs

Keywords:

nonexistence results; a priori bounds; blow up rate; decay rate; periodic orbits; nodal radial solutions; sign-changing solutions

Full Text: DOI

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