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Fibered knots and potential counterexamples to the property 2R and slice-ribbon conjectures. (English) Zbl 1214.57008

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The Property R theorem states that the 0-framed surgery on any non-trivial knot in S^3 does not give $S^1 \times S^2$, where S^k denotes the standard k-dimensional sphere [*D. Gabai*, J. Differ. Geom. 26, 479–536 (1987; Zbl 0639.57008)]. Problem 1.82 in (the new version of) Kirby's problem list [Problems in low-dimensional topology. (Edited by Rob Kirby). Kazez, William H. (ed.), Geometric topology. 1993 Georgia international topology conference, August 2–13, 1993, Athens, GA, USA. Providence, RI: American Mathematical Society. AMS/IP Stud. Adv. Math. 2 (pt.2), 35–473 (1997; Zbl 0888.57014)] conjectures that if surgery on an n-component link in S^3 gives the connected sum of n copies of $S^1 \times S^2$ then the link would become the 0-framed unlink after handle slides (The Generalized Property R Conjecture). The authors propose the following conjecture (Property nR Conjecture) for knots: any knot in S^3 cannot be a component of an n-component counterexample to the Generalized Property R Conjecture. Therefore the Generalized Property R Conjecture states that the Property nR Conjecture is true for all $n \geq 1$.

In the paper under review the authors give potential counterexamples to the Property 2R Conjecture.

The authors first prove that a counterexample to the Property 2R Conjecture with smallest genus is not fibered by showing that any counterexample to the Generalized Property R Conjecture with a fibered component gives another counterexample with smaller genus. They also show that the monodromy of a fibered counterexample has strong restrictions. In particular it is shown that for the square knot Q, the connected sum of the trefoil and its mirror image, if $Q \cup V$ gives a counterexample to the Generalized Property R Conjecture then the restrictions become simple enough to enumerate all such V.

Among these the authors study the link $L_{n,1}$ that consists of Q and the connected sum of the $(n, n+1)$-torus knot and its mirror image. By using four-dimensional techniques it is shown that if the presentation $\langle x, y \mid yxy = xyx, x^{n+1} = y^n \rangle$ of the trivial group gives a counterexample to the Andrews–Curtis conjecture [*J. J. Andrews* and *M. L. Curtis*, Proc. Am. Math. Soc. 16, 192–195 (1965; Zbl 0131.38301)], then $L_{n,1}$ is a counterexample to the Generalized Property R Conjecture.

These examples also give slice knots that are not known to be ribbon giving potential counterexamples to the slice-ribbon problem (Problem 1.33 in Kirby's problem list [loc.cit.]). Here a knot in $S^3 = \partial B^4$ is called slice if it bounds a smooth disk in B^4 and is called ribbon if one can choose such a disk so that it has no local maxima with respect to the radial function.

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MSC:

57M25	Knots and links in the 3-sphere (MSC2010)
57N70	Cobordism and concordance in topological manifolds
57M20	Two-dimensional complexes (manifolds) (MSC2010)
57R65	Surgery and handlebodies
20F05	Generators, relations, and presentations of groups

Keywords:

property R; slice-ribbon conjecture; Andrews–Curtis conjecture

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