Yang, Dachun; **Yang, Dongyong**; **Zhou, Yuan**

Localized Morrey-Campanato spaces on metric measure spaces and applications to Schrödinger operators. (English) Zbl 1214.46019

Nagoya Math. J. 198, 77-119 (2010).

On a space of homogeneous type in the sense of Coifman and Weiss, the authors define and analyse localized Morrey-Campanato and Morrey-Campanato-BLO spaces. They prove the boundedness of radial and Poisson maximal functions between these spaces, as well as the boundedness of the Littlewood-Paley g-function. These results are then applied to prove estimates on the one-parameter semigroups generated by (possibly degenerate) Schrödinger operators on $\mathbb{R}^d$, on Heisenberg groups, and on connected and simply connected nilpotent Lie groups.

Reviewer: Nils Ackermann (Mexico City)

MSC:

46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
35J10 Schrödinger operator, Schrödinger equation
47D06 One-parameter semigroups and linear evolution equations
42B25 Maximal functions, Littlewood-Paley theory

Keywords:

spaces of homogeneous type; maximal function; Schrödinger operator; localized Morrey-Campanato space; operator semigroup

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