Bouche, Daniel; Ghidaglia, Jean-Michel; Pascal, Frédéric P.

Theoretical analysis of the upwind finite volume scheme on the counter-example of Peterson.

(English) Zbl 1213.65123
ESAIM, Math. Model. Numer. Anal. 44, No. 6, 1279-1293 (2010).

The authors consider the upwind finite volume method to approximate an advection problem with a constant velocity in two dimension. By numerous methods, geometric paths counting, recursion, Fourier analysis, generating function, matrix computation, the authors established closed form expressions of an upper bound of the geometric corrector, which has been introduced by the authors some years ago, for the initial (square) *T. E. Peterson* [SIAM J. Numer. Anal. 28, No. 1, 133–140 (1991; Zbl 0729.65085)] and related (semi infinite, triangular) domains for oblique advection velocity. The most convenient expression appears to be a weighted sum of binomial coefficients. As a result, an explicit upper bound for the geometric corrector proportional to h and θ is proved, where θ is the angle of the advection velocity with the vertical. Therefore, the L^∞ norm of the corrector is of order h for a non vertical advection direction . As a consequence, the upwind scheme on initial (square) Peterson mesh and on related (triangular, semi–infinite) meshes is therefore of order h for a non vertical advection direction.

Reviewer: Abdallah Bradji (Annaba)

MSC:

65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs

Keywords:

advection problem with a constant velocity; theoretical analysis; counter-example of Peterson; upwind finite volume; geometric corrector; two dimension

Full Text: DOI EuDML

References:

[1] D. Bouche, J.-M. Ghidaglia and F. Pascal, Error estimate and the geometric corrector for the upwind finite volume method applied to the linear advection equation. SIAM J. Numer. Anal.43 (2005) 578-603. · Zbl 1094.65089 · doi:10.1137/040605941

[2] D. Bouche, J.-M. Ghidaglia and F. Pascal, An optimal a priori error analysis of the finite volume method for linear convection problems, in Finite volumes for complex applications IV, Problems and perspectives , F. Benkhaldoun, D. Ouazar and S. Raghay Eds., Hermes Science publishing, London, UK (2005) 225-236.

[3] B. Cockburn, P.-A. Gremaud and J.X. Yang, A priori error estimates for numerical methods for scalar conservation laws. III: Multidimensional flux-splitting monotone schemes on non-cartesian grids. SIAM J. Numer. Anal.35 (1998) 1775-1803. · Zbl 0909.65058 · doi:10.1137/S0036142997316165

[4] L. Comtet, Advanced combinatorics - The art of finite and infinite expansions. D. Reidel Publishing Co., Dordrecht, The Netherlands (1974). · Zbl 0283.05001

[5] F. Delarue and F. Lagoutière, Probabilistic analysis of the upwind scheme for transport equations. Arch. Ration. Mech. Anal. (to appear). Zbl1230.65008 · Zbl 1230.65008 · doi:10.1007/s00205-010-0322-x

[6] B. Després, An explicit a priori estimate for a finite volume approximation of linear advection on non-cartesian grids. SIAM J. Numer. Anal.42 (2004) 484-504. Zbl1127.65322 · Zbl 1127.65322 · doi:10.1137/S0036142901394558

[7] B. Després, Lax theorem and finite volume schemes. Math. Comp.73 (2004) 1203-1234. · Zbl 1053.65073 · doi:10.1090/S0025-5718-03-01618-1

[8] G.P. Egorychev, Integral representation and the computation of combinatorial sums, Translations of Mathematical Monographs59. American Mathematical Society, Providence, USA (1984). [Translated from the Russian by H.H. McFadden, Translation edited by Lev J. Leifman.] Zbl0524.05001 · Zbl 0524.05001

[9] R. Eymard, T. Gallouët and R. Herbin, Finite volume methods, in Handbook of Numerical Analysis7, P.-A. Ciarlet and J.-L. Lions Eds., North-Holland (2000) 713-1020. · Zbl 0981.65095

[10] W. Feller, An introduction to probability theory and its applicationsI. Third edition, John Wiley & Sons Inc., New York, USA (1968). Zbl0155.23101 · Zbl 0155.23101

[11] S. Karlin, A first course in stochastic processes. Academic Press, New York, USA (1966). · Zbl 0139.33804

[12] D. Kröner, Numerical schemes for conservation laws. Wiley-Teubner Series Advances in Numerical Mathematics, Chichester: Wiley (1997). · Zbl 0872.76001

[13] V. Lakshmikantham and D. Trigiante, Theory of difference equations: numerical methods and applications, 2nd edition, Monographs and Textbooks in Pure and Applied Mathematics251. Marcel Dekker Inc., New York, USA (2002). · Zbl 1014.39001

[14] T.A. Manteuffel and A.B. White, Jr., The numerical solution of second order boundary value problems on nonuniform meshes. Math. Comput. 47 (1986) 511-535. Zbl0635.65092 · Zbl 0635.65092 · doi:10.2307/2008170

[15] B. Merlet, l\infty and l2 error estimate for a finite volume approximation of linear advection. SIAM J. Numer. Anal.46 (2009) 124-150. Zbl1171.35008 · Zbl 1171.35008 · doi:10.1137/060664057

[16] B. Merlet and J. Vovelle, Error estimate for the finite volume scheme applied to the advection equation. Numer. Math.106 (2007) 129-155. · Zbl 1116.35089 · doi:10.1007/s00211-006-0053-y

[17] F. Pascal, On supra-convergence of the finite volume method. ESAIM: Proc.18 (2007) 38-47. · Zbl 1388.76186

[18] T.E. Peterson, A note on the convergence of the discontinuous Galerkin method for a scalar hyperbolic equation. SIAM J. Numer. Anal.28 (1991) 133-140. · Zbl 0729.65085 · doi:10.1137/0728006

[19] M. Renault, Lost (and found) in translation, André's actual method and its application to the generalized ballot problem. Amer. Math. Monthly115 (2008) 358-363. · Zbl 1142.60004

[20] A. Tikhonov and A. Samarskij, Homogeneous difference schemes on non-uniform nets. U.S.S.R. Comput. Math. Math. Phys. 1963 (1964) 927-953. · Zbl 0128.36702 · doi:10.1016/0041-5553(63)90505-6

[21] J.-P. Vila and P. Villedieu, Convergence of an explicit finite volume scheme for first order symmetric systems. Numer. Math.94 (2003) 573-602. Zbl1030.65110 · Zbl 1030.65110 · doi:10.1007/s00211-002-0396-y

[22] J. Vovelle, Convergence of finite volume monotone schemes for scalar conservation laws on bounded domains. Numer. Math. 90 (2002) 563-596. Zbl1007.65066 · Zbl 1007.65066 · doi:10.1007/s002110100307

[23] B. Wendroff and A.B. White, Jr., Some supraconvergent schemes for hyperbolic equations on irregular grids, in Nonlinear hyperbolic equations - Theory, computation methods, and applications (Aachen, 1988), Notes Numer. Fluid Mech.24, Vieweg, Braunschweig, Germany (1989) 671-677. Zbl0674.65067 · Zbl 0674.65067

[24] B. Wendroff and A.B. White, Jr., A supraconvergent scheme for nonlinear hyperbolic systems. Comput. Math. Appl.18 (1989) 761-767. Zbl0683.65078 · Zbl 0683.65078 · doi:10.1016/0898-1221(89)90232-0

[25] H.S. Wilf, generatingfunctionology. Third edition, A K Peters Ltd., Wellesley, USA (2006).