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Analytical solutions for vibrating fractal composite rods and beams. (English) Zbl 1211.74115

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Summary: Fractals have the potential to describe complex microstructures but presently no solution methodologies exist for the prediction of deformation on transiently deforming fractal structures. This is achieved in this paper with the development of analytical solutions on vibrating composite rods and beams. The fractals considered are necessarily deterministic and relatively simple in form to demonstrate the solution methodology. The solutions are limited to beams and rods constructed from an idealised-composite material consisting of relatively large rigid particles embedded in an infinitely thin pliable matrix. Although, as a result, the fractal composite system is not representative of a realistic physical system the methodologies presented do serve to highlight the practical difficulties in using fractals in structural dynamics. Static loading is restricted to spatially invariant axial forces and bending moments as solutions on a unified state of continuum stress are sought which then serve as initial conditions for the vibratory problem. It is demonstrated that measurable displacement is possible on a fractal structure and that finite measures of total, kinetic and strain energy are simultaneously achievable. The approach involves the use of modal analysis to determine modes at natural frequencies that satisfy boundary conditions. These are combined to provide a free vibration solution on a fractal that satisfies the initial conditions in the form of a fractal displacement field.

MSC:

74H45 Vibrations in dynamical problems in solid mechanics
74K10 Rods (beams, columns, shafts, arches, rings, etc.)
74E30 Composite and mixture properties
37N15 Dynamical systems in solid mechanics
74H10 Analytic approximation of solutions (perturbation methods, asymptotic methods, series, etc.) of dynamical problems in solid mechanics

Keywords:

vibration; Cantor dust; analytical solutions; modal analysis

Full Text: DOI

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