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On asymptotic stability of solitary waves in discrete Klein-Gordon equation coupled to a nonlinear oscillator. (English) Zbl 1207.39021
Appl. Anal. 89, No. 9, 1467-1492 (2010).

The author studies the asymptotics of the $U(1)$ -invariant nonlinear discrete Klein-Gordon equation

$$\ddot{\psi}(x, t) = \Delta_L \psi(x, t) - m^2 \psi(x, t) + \delta(x) F(\psi(0, t)), \quad m > 0, \quad x \in \mathbb{Z}.$$

She gives an analysis of the special role of ‘quantum stationary states’ of the form

$$\Psi(x, t) = \Psi_\omega(x) e^{i\omega t}, \quad \omega \in \mathbb{R}, \quad \Psi_\omega = \begin{bmatrix} \psi_\omega \\ i\omega \psi_\omega \end{bmatrix}, \quad \psi_\omega \in l^2(\mathbb{Z}),$$

in which ω and ψ_ω are the solutions of the nonlinear eigenvalue equation

$$-\omega^2 \psi_\omega(x) = \Delta_L \psi_\omega(x) - m^2 \psi_\omega(x) + \delta(x) F(\psi_\omega(0)), \quad x \in \mathbb{Z}.$$

The main result of this paper, which is given in the third section, proves that when the initial state $\Psi_0(x)$ is close to the stable part of the solitary manifold, the following asymptotics hold

$$\Psi(x, t) = \Psi_{\omega_\pm}(x) e^{i\omega_\pm t} + W(t) \Phi_\pm + r_\pm(t), \quad t \rightarrow \pm\infty,$$

where $W(t)$ is the dynamical group of the free Klein-Gordon equation, Φ_\pm are the corresponding asymptotic states, and $\|r_\pm(t)\|_{l^2 \oplus l^2} = \mathcal{O}(|t|^{-1/2})$.

Section 4 to Section 9 contain the proof of the main theorem. The author first summarizes the main properties of the linearized dynamics and then set up the time decay for the linearized equation in continuous spectrum. Moreover, she obtains the modulation equations for the parameters of the soliton. After proving the decay of the transversal component, the main theorem is proved finally.

Reviewer: [Fei Xue \(Hartford\)](#)

MSC:

- [39A30](#) Stability theory for difference equations
- [39A14](#) Partial difference equations
- [39A12](#) Discrete version of topics in analysis
- [35Q40](#) PDEs in connection with quantum mechanics
- [81Q05](#) Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
- [37K10](#) Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)

Cited in 4 Documents

Keywords:

long-time asymptotics; discrete Klein-Gordon equation; nonlinear oscillator; solitary wave; nonlinear eigenvalue equation

Full Text: [DOI](#)

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