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Linear stability analysis for the differentially heated rotating annulus. (English)

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Summary: We use linear stability analysis to approximate the axisymmetric to nonaxisymmetric transition in the differentially heated rotating annulus. We study an accurate mathematical model that uses the Navier-Stokes equations in the Boussinesq approximation. The steady axisymmetric solution satisfies a two-dimensional partial differential boundary value problem. It is not possible to compute the solution analytically, and thus, numerical methods are used. The eigenvalues are also given by a two-dimensional partial differential problem, and are approximated using the matrix eigenvalue problem that results from discretizing the linear part of the appropriate equations. A comparison is made with experimental results. It is shown that the predictions using linear stability analysis accurately reproduce many of the experimental observations. Of particular interest is that the analysis predicts cusping of the axisymmetric to nonaxisymmetric transition curve at wave number transitions, and the wave number maximum along the lower part of the axisymmetric to nonaxisymmetric transition curve is accurately determined. The correspondence between theoretical and experimental results validates the numerical approximations as well as the application of linear stability analysis. A linear stability analysis is also performed with the effects of centrifugal buoyancy neglected. Along the lower part of the transition curve, the results are significantly qualitatively and quantitatively different than when the centrifugal effects are considered. In particular, the results indicate that the centrifugal buoyancy is the cause of the observation of a wave number maximum along the transition curve, and is the cause of a change in concavity of the transition curve.

MSC:

76E07	Rotation in hydrodynamic stability
76D05	Navier-Stokes equations for incompressible viscous fluids
76E20	Stability and instability of geophysical and astrophysical flows
76U05	General theory of rotating fluids

Keywords:

differentially heated rotating fluid experiment; axisymmetric to nonaxisymmetric transition; numerical computation of eigenvalues

Software:

ARPACK

Full Text: DOI

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