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A meshless local boundary integral equation method for two-dimensional steady elliptic problems. (English) Zbl 1206.74023

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Summary: A novel meshless local boundary integral equation (LBIE) method is proposed for the numerical solution of two-dimensional steady elliptic problems, such as heat conduction, electrostatics or linear elasticity. The domain is discretized by a distribution of boundary and internal nodes. From this nodal points' cloud a "background" mesh is created by a triangulation algorithm. A local form of the singular boundary integral equation of the conventional boundary element method is adopted. Its local form is derived by considering a local domain of each node, comprising by the union of neighboring "background" triangles. Therefore, the boundary shape of this local domain is a polygonal closed line. A combination of interpolation schemes is taken into account. Interpolation of boundary unknown field variables is accomplished through boundary element shape functions. On the other hand, the radial basis point interpolation function method is employed for interpolating the unknown interior fields. Essential boundary conditions are imposed directly due to the Kronecker delta-function property of the boundary element interpolation functions. After the numerical evaluation of all boundary integrals, a banded stiffness matrix is constructed, as in the finite element method. Several potential and elastostatic benchmark problems in two dimensions are solved numerically. The proposed meshless LBIE method is also compared with other numerical methods in order to demonstrate its efficiency, accuracy and convergence.

MSC:

74S30 Other numerical methods in solid mechanics (MSC2010)
74S15 Boundary element methods applied to problems in solid mechanics

Keywords:

triangulation algorithm; Green function; convergence; Kronecker delta-function property

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