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SLE and the free field: Partition functions and couplings. (English) Zbl 1204.60079
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The author studies some relations between random curves in planar simply connected domains (Schramm-Loewner Evolutions) and the massless (Euclidean) free field in such a domain. Identities of partition functions between different versions of Schramm-Loewner Evolutions and the free field with appropriate boundary conditions are established. This involves ζ -regularization and the Polyakov-Alvarez conformal anomaly formula. The author proceeds with a construction of couplings of Schramm-Loewner Evolutions with the free field, showing that, in a precise sense, chordal Schramm-Loewner Evolutions is the solution of a stochastic “differential” equation driven by the free field.

Reviewer: [Anatoliy Pogorui \(Zhytomyr\)](#)

MSC:

[60J67](#) Stochastic (Schramm-)Loewner evolution (SLE)
[60G17](#) Sample path properties
[60K35](#) Interacting random processes; statistical mechanics type models; percolation theory

Cited in **76** Documents

Keywords:

[Schramm-Loewner Evolution](#); [massless free field](#); [coupling](#); [Polyakov-Alvarez formula](#)

Full Text: [DOI](#)

References:

- [1] Orlando Alvarez, Theory of strings with boundaries: fluctuations, topology and quantum geometry, Nuclear Phys. B 216 (1983), no. 1, 125 – 184. · [doi:10.1016/0550-3213\(83\)90490-X](#)
- [2] Michel Bauer and Denis Bernard, 2D growth processes: SLE and Loewner chains, Phys. Rep. 432 (2006), no. 3-4, 115 – 221. · [doi:10.1016/j.physrep.2006.06.002](#)
- [3] Vincent Beffara, Hausdorff dimensions for ζ -regularization, Ann. Probab. 32 (2004), no. 3B, 2606 – 2629. · [Zbl 1055.60036](#) · [doi:10.1214/009117904000000072](#)
- [4] Federico Camia and Charles M. Newman, Two-dimensional critical percolation: the full scaling limit, Comm. Math. Phys. 268 (2006), no. 1, 1 – 38. · [Zbl 1117.60086](#) · [doi:10.1007/s00220-006-0086-1](#)
- [5] Giuseppe Da Prato and Jerzy Zabczyk, Second order partial differential equations in Hilbert spaces, London Mathematical Society Lecture Note Series, vol. 293, Cambridge University Press, Cambridge, 2002. · [Zbl 1012.35001](#)
- [6] Julien Dubédat, ζ -regularization martingales and duality, Ann. Probab. 33 (2005), no. 1, 223 – 243. · [Zbl 1096.60037](#) · [doi:10.1214/009117904000000793](#)
- [7] Julien Dubédat, Euler integrals for commuting SLEs, J. Stat. Phys. 123 (2006), no. 6, 1183 – 1218. · [Zbl 1113.82064](#) · [doi:10.1007/s10955-006-9132-9](#)
- [8] Julien Dubédat, Excursion decompositions for SLE and Watts’ crossing formula, Probab. Theory Related Fields 134 (2006), no. 3, 453 – 488. · [Zbl 1112.60032](#) · [doi:10.1007/s00440-005-0446-3](#)
- [9] Julien Dubédat, Commutation relations for Schramm-Loewner evolutions, Comm. Pure Appl. Math. 60 (2007), no. 12, 1792 – 1847. · [Zbl 1137.82009](#) · [doi:10.1002/cpa.20191](#)
- [10] J. Dubédat. Duality of Schramm-Loewner Evolutions. To appear, Ann. Sci. Ecole Normale Supérieure; arXiv:math.PR/0711.1884, 2007.
- [11] J. Dubédat. SLE partition functions, ζ -regularization and Virasoro representations. in preparation, 2007.
- [12] R. Friedrich and J. Kalkkinen, On conformal field theory and stochastic Loewner evolution, Nuclear Phys. B 687 (2004), no. 3, 279 – 302. · [Zbl 1149.81352](#) · [doi:10.1016/j.nuclphysb.2004.03.025](#)
- [13] Krzysztof Gawędź, Lectures on conformal field theory, Quantum fields and strings: a course for mathematicians, Vol. 1, 2 (Princeton, NJ, 1996/1997) Amer. Math. Soc., Providence, RI, 1999, pp. 727 – 805. · [Zbl 1170.81430](#)
- [14] James Glimm and Arthur Jaffe, Quantum physics, 2nd ed., Springer-Verlag, New York, 1987. A functional integral point of view. · [Zbl 0461.46051](#)

- [15] Svante Janson, Gaussian Hilbert spaces, Cambridge Tracts in Mathematics, vol. 129, Cambridge University Press, Cambridge, 1997. · [Zbl 0887.60009](#)
- [16] Richard Kenyon, Conformal invariance of domino tiling, *Ann. Probab.* 28 (2000), no. 2, 759 – 795. · [Zbl 1043.52014](#) · [doi:10.1214/aop/1019160260](#)
- [17] Richard Kenyon, Dominos and the Gaussian free field, *Ann. Probab.* 29 (2001), no. 3, 1128 – 1137. · [Zbl 1034.82021](#) · [doi:10.1214/aop/1015345599](#)
- [18] R. Kenyon and D. Wilson. Boundary Partitions in Trees and Dimers. To appear, *Trans. Amer. Math. Soc.*; preprint, arXiv:math.PR/0608422, 2006.
- [19] Richard W. Kenyon, James G. Propp, and David B. Wilson, Trees and matchings, *Electron. J. Combin.* 7 (2000), Research Paper 25, 34. · [Zbl 0939.05066](#)
- [20] M. Kontsevich. SLE, CFT, and phase boundaries. Arbeitstagung 2003, preprint, MPI 2003 (60).
- [21] M. Kontsevich and Y. Suhov, On Malliavin measures, SLE, and CFT, *Tr. Mat. Inst. Steklova* 258 (2007), no. Anal. i Osob. Ch. 1, 107 – 153; English transl., *Proc. Steklov Inst. Math.* 258 (2007), no. 1, 100 – 146. · [Zbl 1155.81367](#) · [doi:10.1134/S0081543807030108](#)
- [22] Gregory Lawler, Oded Schramm, and Wendelin Werner, Conformal restriction: the chordal case, *J. Amer. Math. Soc.* 16 (2003), no. 4, 917 – 955. · [Zbl 1030.60096](#)
- [23] Gregory F. Lawler, Conformally invariant processes in the plane, *Mathematical Surveys and Monographs*, vol. 114, American Mathematical Society, Providence, RI, 2005. · [Zbl 1074.60002](#)
- [24] Gregory F. Lawler, Oded Schramm, and Wendelin Werner, Conformal invariance of planar loop-erased random walks and uniform spanning trees, *Ann. Probab.* 32 (2004), no. 1B, 939 – 995. · [Zbl 1126.82011](#) · [doi:10.1214/aop/1079021469](#)
- [25] Gregory F. Lawler and José A. Trujillo Ferreras, Random walk loop soup, *Trans. Amer. Math. Soc.* 359 (2007), no. 2, 767 – 787. · [Zbl 1120.60037](#)
- [26] Gregory F. Lawler and Wendelin Werner, The Brownian loop soup, *Probab. Theory Related Fields* 128 (2004), no. 4, 565 – 588. · [Zbl 04576228](#) · [doi:10.1007/s00440-003-0319-6](#)
- [27] Y. Le Jan. Markov loops, determinants and Gaussian fields. arxiv:math.PR/0612112, 2006.
- [28] B. Osgood, R. Phillips, and P. Sarnak, Extremals of determinants of Laplacians, *J. Funct. Anal.* 80 (1988), no. 1, 148 – 211. · [Zbl 0653.53022](#) · [doi:10.1016/0022-1236\(88\)90070-5](#)
- [29] A. M. Polyakov, Quantum geometry of bosonic strings, *Phys. Lett. B* 103 (1981), no. 3, 207 – 210. , [https://doi.org/10.1016/0370-2693\(81\)90743-7](https://doi.org/10.1016/0370-2693(81)90743-7) A. M. Polyakov, Quantum geometry of fermionic strings, *Phys. Lett. B* 103 (1981), no. 3, 211 – 213. · [doi:10.1016/0370-2693\(81\)90744-9](#)
- [30] Daniel Revuz and Marc Yor, Continuous martingales and Brownian motion, 3rd ed., *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*, vol. 293, Springer-Verlag, Berlin, 1999. · [Zbl 0917.60006](#)
- [31] Steffen Rohde and Oded Schramm, Basic properties of SLE, *Ann. of Math. (2)* 161 (2005), no. 2, 883 – 924. · [Zbl 1081.60069](#) · [doi:10.4007/annals.2005.161.883](#)
- [32] Oded Schramm, Scaling limits of loop-erased random walks and uniform spanning trees, *Israel J. Math.* 118 (2000), 221 – 288. · [Zbl 0968.60093](#) · [doi:10.1007/BF02803524](#)
- [33] O. Schramm and S. Sheffield. Contour lines of the two-dimensional discrete Gaussian free field. *Acta Math.*, 202(1):21-137, 2009. · [Zbl 1210.60051](#)
- [34] O. Schramm and S. Sheffield. In preparation. 2007.
- [35] S. Sheffield. Exploration trees and conformal loop ensembles. preprint, arXiv:math.PR/ 0609167, 2006.
- [36] Scott Sheffield, Gaussian free fields for mathematicians, *Probab. Theory Related Fields* 139 (2007), no. 3-4, 521 – 541. · [Zbl 1132.60072](#) · [doi:10.1007/s00440-006-0050-1](#)
- [37] Barry Simon, *The $\varphi(\varphi)_2$ Euclidean (quantum) field theory*, Princeton University Press, Princeton, N.J., 1974. Princeton Series in Physics. · [Zbl 1175.81146](#)
- [38] Barry Simon, Trace ideals and their applications, 2nd ed., *Mathematical Surveys and Monographs*, vol. 120, American Mathematical Society, Providence, RI, 2005. · [Zbl 1074.47001](#)
- [39] Hidenori Sonoda, Functional determinants on punctured Riemann surfaces and their application to string theory, *Nuclear Phys. B* 294 (1987), no. 1, 157 – 192. · [doi:10.1016/0550-3213\(87\)90578-5](#)
- [40] Wendelin Werner, Random planar curves and Schramm-Loewner evolutions, *Lectures on probability theory and statistics*, *Lecture Notes in Math.*, vol. 1840, Springer, Berlin, 2004, pp. 107 – 195. · [Zbl 1057.60078](#) · [doi:10.1007/978-3-540-39982-7_2](#)
- [41] Dapeng Zhan, Duality of chordal SLE, *Invent. Math.* 174 (2008), no. 2, 309 – 353. · [Zbl 1158.60047](#) · [doi:10.1007/s00222-008-0132-z](#)
- [42] Dapeng Zhan, Reversibility of chordal SLE, *Ann. Probab.* 36 (2008), no. 4, 1472 – 1494. · [Zbl 1157.60051](#) · [doi:10.1214/07-AOP366](#)

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