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On the quadrature and weak form choices in collocation type discontinuous Galerkin spectral element methods. (English) [Zbl 1203.65199](#)
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Summary: We examine four nodal versions of tensor product discontinuous Galerkin spectral element approximations to systems of conservation laws for quadrilateral or hexahedral meshes. They arise from the two choices of Gauss or Gauss-Lobatto quadrature and integrate by parts once (I) or twice (II) formulations of the discontinuous Galerkin method. We show that the two formulations are in fact algebraically equivalent with either Gauss or Gauss-Lobatto quadratures when global polynomial interpolations are used to approximate the solutions and fluxes within the elements. Numerical experiments confirm the equivalence of the approximations and indicate that using Gauss quadrature with integration by parts once is the most efficient of the four approximations.

MSC:

- 65M70** Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs

Cited in **1** Review
Cited in **51** Documents

Keywords:

spectral element; discontinuous Galerkin; collocation

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