Optimal error estimates of the Legendre tau method for second-order differential equations.

Summary: We prove that the Legendre tau method has the optimal rate of convergence in $L^2$-norm, $H^1$-norm and $H^2$-norm for one-dimensional second-order steady differential equations with three kinds of boundary conditions and in $C([0,T];L^2(I))$-norm for the corresponding evolution equation with the Dirichlet boundary condition. For the generalized Burgers equation, we develop a Legendre tau-Chebyshev collocation method, which can also be optimally convergent in $C([0,T];L^2(I))$-norm. Finally, we give some numerical examples.

MSC:
65L60 Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations
65L70 Error bounds for numerical methods for ordinary differential equations
65M15 Error bounds for initial value and initial-boundary value problems involving PDEs
65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

Keywords:
tau method; optimal error estimate; second-order differential equation

Full Text: DOI

References: