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On an extended clarifier-thickener model with singular source and sink terms. (English)

Zbl 1201.35130

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Summary: A one-dimensional model of clarifier-thickener units in engineering applications can be expressed as a conservation law with a flux that is discontinuous with respect to the spatial variable. This model also includes a singular feed source. In this paper, the clarifier-thickener model studied in a previous paper [Numer. Math. 97, 25–65 (2004; Zbl 1053.76047)] is extended by a singular sink through which material is extracted from the unit. A difficulty is that in contrast to the singular source, the sink term cannot be incorporated into the flux function; rather, the sink is represented by a new non-conservative transport term. To focus on the new analytical difficulties arising due to this non-conservative term, a reduced problem is formulated, which contains the new sink term of the extended clarifier-thickener model, but not the source term and flux discontinuities. The paper is concerned with numerical methods for both models (extended and reduced) and with the well-posedness analysis for the reduced problem. For the reduced problem, a definition of entropy solutions, based on Kruškov-type entropy functions and fluxes, is provided. Jump conditions are derived and uniqueness of the entropy solution is shown. Existence of an entropy solution is shown by proving convergence of a monotone difference scheme. Two variants of the numerical scheme are introduced. Numerical examples illustrate that all three variants converge to the entropy solution, but introduce different amounts of numerical diffusion.

MSC:

35L60 First-order nonlinear hyperbolic equations

76M20 Finite difference methods applied to problems in fluid mechanics

76T20 Suspensions

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

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