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Theory of p -adic distributions. Linear and nonlinear models. (English) Zbl 1198.46001

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For a prime number p , the p -adic field \mathbb{Q}_p is defined as the completion of the field \mathbb{Q} of rational numbers with respect to the p -adic valuation $|r|_p = p^{-\alpha}$, where $r = p^\alpha z_1/z_2$, with $z_1, z_2 \in \mathbb{Z}$ relatively prime with p . What makes the p -adic analysis differ drastically from the classical one (meaning the analysis over the field \mathbb{R} or \mathbb{C}) is the ultrametric inequality, $|r + q|_p \leq \max\{|r|_p, |q|_p\}$, satisfied by the p -adic valuation $|\cdot|_p$. In the beginning, p -adic analysis developed mainly in connection with some problems in algebraic number theory, but with time it turned into a well-developed discipline of its own. Recently, deep applications of p -adic analysis in quantum mechanics and quantum field theory, quantum gravity and cosmology, string theory, as well as in cognitive sciences, psychology, genetics information science, have emerged.

As the authors mention, there are two approaches in studying p -adic functions: as functions from subsets of \mathbb{Q}_p^n to \mathbb{C} , or to \mathbb{Q}_p . The first approach was proposed in the book by *V. S. Vladimirov, I. V. Volovich* and *E. I. Zelenov* [*" p -adic analysis and mathematical physics"* (Singapore: World Scientific) (1994; [Zbl 0812.46076](#))], while the authors of the present book consider \mathbb{Q}_p -valued functions. The aim of this book is to develop distribution theory within this framework, with applications to p -adic pseudo-differential operators and pseudo-differential equations.

In order to make it self-contained, the authors expose (with full proofs) in the first three chapters, 1. p -adic numbers, 2. p -adic functions and 3. p -adic integration theory, the basics of p -adic fields and p -adic analysis. The study of p -adic distributions starts in Chapter 5, p -adic distributions, and continues in Chapter 6, the theory of associated and quasi-associated p -adic distributions, and 7, p -adic Lizorkin spaces of test functions and distributions. A consistent chapter (70 pages), 8, The theory of p -adic wavelets, is devoted to the presentation of this basic tool, which, as in the classical case, plays a key role in the applications of p -adic analysis. These first eight chapters of the book contain material that can be found also in other monographs on p -adic analysis.

The remaining, more advanced, nine chapters are based on the original results of the authors. The chapters 9, Pseudo-differential operators on Lizorkin spaces, 10, Pseudo-differential equations, 11, A p -adic Schrödinger-type operator with point interactions, 12, Distributional asymptotics and p -adic Tauberian theorems, 13, Asymptotics of the p -adic singular Fourier integrals, 14, Nonlinear theories of p -adic generalized functions, treat new developments in p -adic harmonic analysis, in p -adic pseudo-differential operators and equations, p -adic Tauberian theorems, etc.

Extensively based on the original results of the authors, this valuable monograph will be of interest to researchers in p -adic analysis as well as to those interested in its applications to mathematical physics, biology or to other areas. Its extended introductory part makes it also accessible to graduate students desiring to be quickly acquainted with the methods of p -adic analysis.

Reviewer: [Stefan Cobzaş \(Cluj-Napoca\)](#)

MSC:

- 46-02 Research exposition (monographs, survey articles) pertaining to functional analysis
- 46S10 Functional analysis over fields other than \mathbb{R} or \mathbb{C} or the quaternions; non-Archimedean functional analysis
- 46F05 Topological linear spaces of test functions, distributions and ultradistributions
- 46F30 Generalized functions for nonlinear analysis (Rosinger, Colombeau, nonstandard, etc.)
- 42C40 Nontrigonometric harmonic analysis involving wavelets and other special systems
- 35S05 Pseudodifferential operators as generalizations of partial differential operators
- 47G30 Pseudodifferential operators

Cited in **6** Reviews
Cited in **80** Documents

Keywords:

p-adic valued field; *p*-adic analytic functions; *p*-adic distributions; *p*-adic wavelets; pseudo-differential operators; pseudo-differential equations