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Construction and asymptotic behavior of multi-peak solutions to the Gierer-Meinhardt system with saturation. (English) Zbl 1197.35023

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The authors consider existence and multiplicity of stationary states for the Gierer-Meinhard system with saturation:

$$\begin{cases} A_t = \varepsilon^2 \Delta A - A + \dfrac{A^2}{H(1+kA^2)}, & \text{in } \Omega \times (0,\infty),\\ \tau H_t = D\Delta H - H + A^2, & \text{in } \Omega \times (0,\infty),\\ \dfrac{\partial A}{\partial \nu} = \dfrac{\partial H}{\partial \nu} = 0, & \text{on } \partial\Omega \times (0,\infty),\\ A>0,\ H>0, & \text{in } \Omega \times (0,\infty). \end{cases} \tag{1}$$

Here $\Omega \subseteq \mathbb{R}^N$ is a bounded smooth domain, $2 \le N \le 5$, $\tau \ge 0$, and $\varepsilon, k > 0$. Moreover, they assume that Ω is rotationally symmetric with respect to the x_N-axis, and that $k = k(\varepsilon)$ and ε have the dependence $\lim_{\varepsilon\to 0} 4k\varepsilon^{-2N}|\Omega|^2 = k_0$, for some $k_0 \in [0,\infty)$ that is sufficiently small.

Fixing a subset $\{P_1, P_2, \dots, P_m\}$ of the finite set of intersections of the x_N-axis with $\partial\Omega$ the following result is obtained: If ε is sufficiently small and D sufficiently large then there exists a stationary solution to (1) that has its mass concentrated near the points P_i. These spikes are individually approximated, asymptotically as $\varepsilon \to 0$ and $D \to \infty$, by the rescaled unique solution of a suitable limit equation posed in $\mathbb{R}^N$.

A key point in the proof is that the symmetry condition on Ω allows to construct a unique symmetric multi-peak solution of a related nonlinear elliptic equation that depends on a suitably defined new parameter δ. Uniqueness in turn leads to continuous dependence of this solution on δ and allows to obtain a multi-peak stationary state for the shadow system (where $D = \infty$). Finally, to obtain a stationary state for the original equation (1) the implicit function theorem is used.

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MSC:

35B25 Singular perturbations in context of PDEs
35J57 Boundary value problems for second-order elliptic systems
35K57 Reaction-diffusion equations
35Q92 PDEs in connection with biology, chemistry and other natural sciences
92C15 Developmental biology, pattern formation
35J60 Nonlinear elliptic equations

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