Sweet, Erik; **Van Gorder, Robert A.**

Analytical solutions to a generalized Drinfel'd-Sokolov equation related to DSSH and KdV6.

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Summary: Analytical solutions to the generalized Drinfel'd-Sokolov (GDS) equations

$$u_t + \alpha_1 u u_x + \beta_1 u_{xxx} + \gamma (v^\delta)_x = 0 \quad \text{and} \quad v_t + \alpha_2 u v_x + \beta_2 v_{xxx} = 0$$

are obtained for various values of the model parameters. In particular, we provide perturbation solutions to illustrate the strong influence of the parameters β_1 and β_2 on the behavior of the solutions. We then consider a Miura-type transform which reduces the gDS equations into a sixth-order nonlinear differential equation under the assumption that $\delta = 1$. Under such a transform the GDS reduces to the sixth-order Drinfel'd-Sokolov-Satsuma-Hirota (DSSH) equation (also known as KdV6) in the very special case $\alpha_1 = -\alpha_2$. The method of homotopy analysis is applied in order to obtain analytical solutions to the resulting equation for arbitrary α_1 and α_2. An error analysis of the obtained approximate analytical solutions is provided.

MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
35B20 Perturbations in context of PDEs
35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs
35A22 Transform methods (e.g., integral transforms) applied to PDEs
35A35 Theoretical approximation in context of PDEs
35C05 Solutions to PDEs in closed form

Keywords:

Drinfel'd-Sokolov equation; analytical solution; nonlinear PDE

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