Abramovich, Dan; Graber, Tom; Vistoli, Angelo
Gromov-Witten theory of Deligne-Mumford stacks. (English) Zbl 1193.14070
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The paper under review is devoted to establishing rigorous foundations to the Gromov-Witten theory of smooth complex Deligne-Mumford stacks with a projective coarse moduli space.

The contents of the paper were announced earlier by the same authors [Contemp. Math. 310, 1–24 (2002; Zbl 1067.14055)] with the aim of giving an algebro-geometric counterpart to the symplectic version of Gromov-Witten theory of orbifolds previously developed by *W. Chen* and *Y. Ruan* [Contemp. Math. 310, 25–85 (2002; Zbl 1091.53058)]. The stack-theoretical framework presented here is expected to be useful also within the symplectic setting.

The classical Gromov-Witten theory of a smooth projective variety X within the algebro-geometric setting relies on the Kontsevich moduli space of stable maps from n-pointed curves of given genus g to X with image class $\beta \in H_2(X,\mathbb{Z})$. To work out the Gromov-Witten theory of an orbifold $\mathcal{X}$ this space is then replaced by a moduli space of maps from orbifold curves to $\mathcal{X}$. Such a space had already been constructed by the first and the third author [J. Am. Math. Soc. 15, No. 1, 27–75 (2002; Zbl 0991.14007)] and later generalized by *M. Olsson* [Duke Math. J. 134, No. 1, 139–164 (2006; Zbl 1114.14002)] and [J. Reine Angew. Math. 603, 55–112 (2007; Zbl 1137.14004)] and goes with the name of moduli stack of twisted stable maps, $\mathcal{K}_{g,n}(\mathcal{X},\beta)$.

There are several technical issues that need to be worked out in order to develop a satisfactory Gromov-Witten theory for $\mathcal{X}$. The existence of the virtual fundamental class for $\mathcal{K}_{g,n}(\mathcal{X},\beta)$ follows without major difficulties from the classical case while evaluation maps are shown to land not in $\mathcal{X}$ but in a new gadget called the *rigidified cyclotomic inertia stack* of $\mathcal{X}$, denoted by $\overline{\mathcal{I}}_\mu(\mathcal{X})$, parametrizing gerbes banded by some μ_r together with a representable morphism to $\mathcal{X}$. Gromov-Witten classes are then naturally defined in $A^*(\overline{\mathcal{I}}_\mu(\mathcal{X}))_\mathbb{Q}$ and differ from the classical case also by a correction term describing the index of the gerbe in $\overline{\mathcal{I}}_\mu(\mathcal{X})$.

Some basic properties of classical Gromov-Witten invariants are then shown to hold also in the orbifold case, being of particular relevance the proof of the WDVV equation, which is Theorem 6.2.1 in the paper.

Reviewer: Margarida Melo (Coimbra)

MSC:

14N35	Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)
14A20	Generalizations (algebraic spaces, stacks)
53D45	Gromov-Witten invariants, quantum cohomology, Frobenius manifolds

Keywords:

Deligne-Mumford stack; Gromov-Witten invariants; twisted curves; cyclotomic inertia stack

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