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An integral structure in quantum cohomology and mirror symmetry for toric orbifolds.

(English) [Zbl 1190.14054](#)

Adv. Math. 222, No. 3, 1016-1079 (2009).

Consider a weak Fano projective toric orbifold \mathcal{X} . The author introduces a $\widehat{\Gamma}$ -integral structure on the quantum D -module of \mathcal{X} , that is an integral structure on the space of flat sections of Dubrovin's connection for \mathcal{X} given by a class

$$\widehat{\Gamma}(T\mathcal{X}) = \prod_{i=1}^{\dim \mathcal{X}} \Gamma(1 + \delta_i),$$

where δ_i 's are Chern roots of \mathcal{X} . The main theorem (Theorem 4.11) states that under some assumptions this integral structure corresponds, modulo Mirror Conjecture, to the natural integral local system on the mirror B-model D -module under the mirror isomorphism. In particular this holds for toric manifolds as assumptions are proven to hold. By assuming the existence of an integral structure, the author gives a natural explanation for the specialization to a root of unity in Y . Ruan's crepant resolution conjecture [in: AMS special session, San Francisco, CA, USA, May 3–4, 2003. Providence, RI: American Mathematical Society (AMS). Contemporary Mathematics 403, 117–126 (2006; [Zbl 1105.14078](#))].

Reviewer: [Victor Przyjalkowski \(Moskva\)](#)

MSC:

14N35 Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebraic-geometric aspects)

53D45 Gromov-Witten invariants, quantum cohomology, Frobenius manifolds

Cited in **3** Reviews
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Keywords:

quantum cohomology; variation of Hodge structures; semi-infinite variation of Hodge structures; mirror symmetry; Landau-Ginzburg model; toric Deligne-Mumford stack; orbifold; orbifold quantum cohomology; Crepant resolution conjecture; Ruan's conjecture; K -theory; McKay correspondence; oscillatory integral; hypergeometric function; GKZ-system; singularity theory; gamma class

Full Text: [DOI](#) [arXiv](#)

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