Guo, Yuxia; **Liu, Jiaquan**
Bifurcation for strongly indefinite functional and applications to Hamiltonian system and noncooperative elliptic system. (English) ⎢Zbl 1187.37024⎢
J. Math. Anal. Appl. 359, No. 1, 28-38 (2009).

This article covers the extension of bifurcation results given in [*K.-C. Chang, Z.-Q. Wang*, J. Fixed Point Theory Appl. 1, No. 2, 195–208 (2007; Zbl 1139.58008)] to the case of strongly indefinite functionals, i.e., where positive and negative eigenspaces of the second differential at the bifurcation point are infinite dimensional.

Consider functionals

$$f_\lambda := \frac{1}{2}((A + B)u, u) - \frac{1}{2}\lambda(Ju, u) + g(u)$$

defined on a real Hilbert space H. Here λ is a real parameter, $(\cdot, \cdot)$ denotes the inner product, A, B and J denote linear, bounded self-adjoint operators in H, B is compact, and $g : H \to \mathbb{R}$ is differentiable and such that $G := g'$ gives a Lipschitz continuous compact nonlinear operator that satisfies $G(u) = o(\|u\|)$ as $u \to 0$.

In earlier work [Nonlinear Anal., Theory Methods Appl. 48, No. 6(A), 831–851 (2002; Zbl 1013.37023)] the authors define critical groups and a degree theory for dynamically isolated critical sets of the negative pseudogradient flow generated by f_λ, via a Galerkin approximation scheme. Using these notions and employing the Maslov index of B with respect to A they formulate two abstract theorems stating the existence of bifurcation from 0 and from ∞ under conditions on the critical groups and indices of critical sets.

These theorems are applied to a periodic Hamiltonian system and to a non-cooperative elliptic system.

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MSC:

37B30	Index theory for dynamical systems, Morse-Conley indices	Cited in **2** Documents
58E07	Variational problems in abstract bifurcation theory in infinite-dimensional spaces	
58E05	Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces	
47H11	Degree theory for nonlinear operators	
37J45	Periodic, homoclinic and heteroclinic orbits; variational methods, degree-theoretic methods (MSC2010)	
35J50	Variational methods for elliptic systems	

Keywords:

strongly indefinite functional; Morse theory; critical group; topological degree; Maslov index; bifurcation

Full Text: DOI

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