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Word equations over graph products. (English) Zbl 1186.20041
Int. J. Algebra Comput. 18, No. 3, 493-533 (2008).

The existence of algorithms recognizing the solvability of equations in a free semigroup was established by G. S. Makanin more than four decades ago and has prompted a wealth of research, well referenced in the paper under review. The authors continue this stream of research by establishing transfer results to graph products (a construction that generalizes both free and direct products) of monoids.

From the summary: “For monoids that satisfy a weak cancellation condition it is shown that the decidability of the existential theory of word equations is preserved under graph products. Furthermore, it is shown that the positive theory of a graph product of groups can be reduced to the positive theories of those factors which commute with all other factors and the existential theories of the remaining factors. Both results also include suitable constraints for the variables. Larger classes of constraints lead in many cases to undecidability results.”

Reviewer: [Herman J. Servatius \(Worcester\)](#)

MSC:

- [20M05](#) Free semigroups, generators and relations, word problems
- [03B25](#) Decidability of theories and sets of sentences
- [20F10](#) Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)
- [20E06](#) Free products of groups, free products with amalgamation, Higman-Neumann-Neumann extensions, and generalizations
- [03D35](#) Undecidability and degrees of sets of sentences
- [68Q70](#) Algebraic theory of languages and automata
- [68Q42](#) Grammars and rewriting systems
- [20F70](#) Algebraic geometry over groups; equations over groups
- [68Q25](#) Analysis of algorithms and problem complexity

Cited in **9** Documents

Keywords:

equations in groups; equations in monoids; logical theories; graph products; decidability; algorithms; solvability of equations; existential theories; positive theories

Full Text: [DOI](#)

References:

- [1] DOI: [10.1007/BF02088289](#) · Zbl [0679.68132](#) · doi:[10.1007/BF02088289](#)
- [2] DOI: [10.1007/BF01895842](#) · Zbl [0381.20027](#) · doi:[10.1007/BF01895842](#)
- [3] DOI: [10.1007/BF01917515](#) · doi:[10.1007/BF01917515](#)
- [4] DOI: [10.1016/0022-4049\(91\)90139-S](#) · Zbl [0749.20006](#) · doi:[10.1016/0022-4049\(91\)90139-S](#)
- [5] DOI: [10.1016/0304-3975\(80\)90037-7](#) · Zbl [0475.03017](#) · doi:[10.1016/0304-3975\(80\)90037-7](#)
- [6] DOI: [10.1007/978-3-663-09367-1](#) · doi:[10.1007/978-3-663-09367-1](#)
- [7] DOI: [10.1007/978-1-4613-9771-7](#) · doi:[10.1007/978-1-4613-9771-7](#)
- [8] DOI: [10.1090/S0002-9947-00-02506-X](#) · Zbl [1029.20018](#) · doi:[10.1090/S0002-9947-00-02506-X](#)
- [9] Büchi J. R., *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 34 pp 337–
- [10] Cartier P., *Lecture Notes in Mathematics* 85, in: *Problèmes Combinatoires de Commutation et Réarrangements* (1969) · Zbl [0186.30101](#) · doi:[10.1007/BFb0079468](#)
- [11] DOI: [10.1016/j.ic.2005.04.002](#) · Zbl [1101.68649](#) · doi:[10.1016/j.ic.2005.04.002](#)
- [12] DOI: [10.1142/S0218196702000870](#) · Zbl [1007.03008](#) · doi:[10.1142/S0218196702000870](#)
- [13] DOI: [10.1007/s00224-003-1110-x](#) · Zbl [1067.03017](#) · doi:[10.1007/s00224-003-1110-x](#)

- [14] DOI: 10.1016/S0304-3975(98)00313-2 · Zbl 0930.68074 · doi:10.1016/S0304-3975(98)00313-2
- [15] DOI: 10.1142/S0218196706003372 · Zbl 1112.03009 · doi:10.1142/S0218196706003372
- [16] DOI: 10.1142/9789814261456 · doi:10.1142/9789814261456
- [17] DOI: 10.1016/0021-8693(87)90010-X · Zbl 0692.05035 · doi:10.1016/0021-8693(87)90010-X
- [18] Durnev V. G., *Sibirsky Matematicheskie Jurnal* 36 pp 1067–
- [19] Epstein D. B. A., *Word Processing in Groups* (1992)
- [20] Feferman S., *Fund. Math.* 47 pp 57–
- [21] DOI: 10.1007/s00233-006-0602-9 · Zbl 1124.20040 · doi:10.1007/s00233-006-0602-9
- [22] DOI: 10.1006/jabr.1995.1010 · Zbl 0831.20032 · doi:10.1006/jabr.1995.1010
- [23] DOI: 10.1017/CBO9780511551574 · doi:10.1017/CBO9780511551574
- [24] Jantzen M., *EATCS Monographs on Theoretical Computer Science* 14 (1988)
- [25] DOI: 10.1142/S0218196707003330 · Zbl 1184.20031 · doi:10.1142/S0218196707003330
- [26] DOI: 10.1016/j.jalgebra.2006.03.033 · Zbl 1110.03020 · doi:10.1016/j.jalgebra.2006.03.033
- [27] DOI: 10.1142/S0218196706003001 · Zbl 1151.03003 · doi:10.1142/S0218196706003001
- [28] DOI: 10.1142/S021819670200122X · Zbl 1044.20024 · doi:10.1142/S021819670200122X
- [29] DOI: 10.1007/978-3-642-61896-3 · doi:10.1007/978-3-642-61896-3
- [30] Makanin G. S., *Math. Sbornik* 103 pp 147–
- [31] Makanin G. S., *Izv. Akad. Nauk SSR, Ser. Math.* 46 pp 1199–
- [32] Makanin G. S., *Izv. Akad. Nauk SSSR, Ser. Mat.* 48 pp 735–
- [33] Marchenkov S. S., *Sibirsky Matematicheskie Jurnal* 23 pp 196–
- [34] DOI: 10.2140/pjm.1964.14.1343 · Zbl 0144.01201 · doi:10.2140/pjm.1964.14.1343
- [35] Meier J., *Geometriae Dedicata* 61 pp 29–
- [36] Merzlyakov Y. I., *Algebra i Logika Sem.* 5 pp 25–
- [37] DOI: 10.1016/0020-0190(88)90049-X · Zbl 0653.03026 · doi:10.1016/0020-0190(88)90049-X
- [38] DOI: 10.2307/1968867 · Zbl 0060.12501 · doi:10.2307/1968867
- [39] Ochmański E., *Bull. Eur. Assoc. Theoret. Comput. Sci. (EATCS)* 27 pp 56–
- [40] Papadimitriou C. H., *Computational Complexity* (1994) · Zbl 0833.68049
- [41] DOI: 10.1145/990308.990312 · Zbl 1192.68372 · doi:10.1145/990308.990312
- [42] M. Presburger, *Comptes Rendus du Premier Congrès des Mathématiciens des Pays Slaves (Warsaw, 1929)* pp. 92–101.
- [43] DOI: 10.2307/2268308 · Zbl 0063.06362 · doi:10.2307/2268308
- [44] Repin N. N., *Voprosy Kibernetiki (Moskva)* 134 pp 167–
- [45] DOI: 10.1007/BF01241140 · Zbl 0845.57002 · doi:10.1007/BF01241140
- [46] DOI: 10.1007/BF00970391 · Zbl 0452.20057 · doi:10.1007/BF00970391
- [47] DOI: 10.1007/BF00969107 · Zbl 0595.03006 · doi:10.1007/BF00969107
- [48] DOI: 10.1016/0304-3975(87)90080-6 · Zbl 0634.68076 · doi:10.1016/0304-3975(87)90080-6
- [49] K. U. Schulz, *Word Equations and Related Topics, Lecture Notes in Computer Science* 572, ed. K. U. Schulz (Springer, 1991) pp. 85–150. · doi:10.1007/3-540-55124-7_4
- [50] Semenov A. L., *Soviet Math. Dokl.* 21 pp 952–
- [51] DOI: 10.1070/SM1983v044n01ABEH000954 · Zbl 0497.20046 · doi:10.1070/SM1983v044n01ABEH000954
- [52] DOI: 10.1007/s002330010075 · Zbl 0992.20042 · doi:10.1007/s002330010075
- [53] DOI: 10.1051/ita:2001103 · Zbl 1019.20028 · doi:10.1051/ita:2001103

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