Martínez-Pedroza, Eduardo

Combination of quasiconvex subgroups of relatively hyperbolic groups. (English)

Zbl 1186.20029
Groups Geom. Dyn. 3, No. 2, 317-342 (2009).

Let G be a group generated by a finite set X and hyperbolic relative to a collection of subgroups $\mathcal{H}$. A subgroup of G is called parabolic if it can be conjugated into one of the subgroups in $\mathcal{H}$. Moreover, a subgroup of G is called a relatively quasiconvex subgroup if it is a quasiconvex subgroup of the coned-off Cayley graph of $(G, X, \mathcal{H})$.

In the paper under review are proved the following main theorems. 1. For any relatively quasiconvex subgroup Q and any maximal parabolic subgroup P of G, there is a constant $C = C(Q, P) \geq 0$ with the following property. If R is a subgroup of P such that (a) $Q \cap P \subset R$, and (b) $d_X(g, 1) \geq C$ for any $g \in R \backslash Q$, then the natural homomorphism $Q *_{Q\cap R} R \to G$ is injective with image a relatively quasiconvex subgroup. Moreover, every parabolic subgroup of $\langle Q \cup R\rangle \subset G$ is either conjugate to a subgroup of Q or a subgroup of R in $\langle Q \cup R\rangle$.

2. For any pair of relatively quasiconvex subgroups Q_1 and Q_2, and any maximal parabolic subgroup P such that $R = Q_1 \cap P = Q_2 \cap P$, there is a constant $C = C(Q_1, Q_2, P) \geq 0$ with the following property. If $h \in P$ is such that (a) $hRh^{-1} = R$, and (b) $d_X(q, 1) \geq C$ for any $g \in RhR$, then the natural homomorphism $Q_1 *_R hQ_2h^{-1} \to G$ is injective and its image is a relatively quasiconvex subgroup. Moreover, every parabolic subgroup of $\langle Q_1 \cup hQ_2h^{-1}\rangle \subset G$ is either conjugate to a subgroup of Q_1 or hQ_2h^{-1} in $\langle Q_1 \cup hQ_2h^{-1}\rangle$. Here d_X denotes a word metric induced by X on G.

Reviewer: Andrzej Szczepański (Gdańsk)

MSC:

20F67	Hyperbolic groups and nonpositively curved groups
20F65	Geometric group theory
20E06	Free products of groups, free products with amalgamation, Higman-Neumann-Neumann extensions, and generalizations
20F05	Generators, relations, and presentations of groups
57M07	Topological methods in group theory
20E07	Subgroup theorems; subgroup growth

Keywords:

coned-off Cayley graphs; relative hyperbolicity; quasiconvex subgroups; combination theorems; parabolic subgroups

Full Text: DOI Link arXiv

References:

[1] I. Agol, D. Groves, J. Manning, Residual finiteness, QCERF, and fillings of hyperbolic groups. Geom. Topol. 13 (2009), 1043-1073. · Zbl 1229.20037 · doi:10.2140/gt.2009.13.1043

[2] E. Alibegović, A combination theorem for relatively hyperbolic groups. Bull. London Math. Soc. 37 (2005), 459-466. · Zbl 1074.57001 · doi:10.1112/S0024609304004059

[3] J. W. Anderson, J. Aramayona, and K. J. Shackleton, An obstruction to the strong rel- ative hyperbolicity of a group. J. Group Theory 10 (2007), 749-756. · Zbl 1188.20041 · doi:10.1515/JGT.2007.054

[4] G. N. Arzhantseva, On quasiconvex subgroups of word hyperbolic groups. Geom. Dedi- cata 87 (2001), 191-208. · Zbl 0994.20036 · doi:10.1023/A:1012040207144

[5] G. Arzhantseva and A. Minasyan, Relatively hyperbolic groups are C -simple. J. Funct. Anal. 243 (2007), 345-351. · Zbl 1115.20034 · doi:10.1016/j.jfa.2006.06.003

[6] M. Baker and D. Cooper, A combination theorem for convex hyperbolic manifolds, with applications to surfaces in 3-manifolds. J. Topology 1 (2008), 603-642. · Zbl 1151.57014 · doi:10.1112/jtopol/jtn013

[7] J. Behrstock, C. Drutu, and L. Mosher, Thick metric spaces, relative hyperbolicity, and quasi-isometric rigidity. Preprint 2005.

[8] M. Bestvina, Questions in geometric group theory. Preprint 2004.

[9] B. H. Bowditch, Relatively hyperbolic groups. Preprint 1999. · Zbl 1259.20052 · doi:10.1142/S0218196712500166

[10] N. Brady, M. Forester, and E. Martinez-Pedroza, Surface subgroups of hyperbolization of groups. In preparation.

[11] D. Calegari, Surface subgroups from homology. Geom. Topol. 12 (2008), 1995-2007. · Zbl 1185.20046 · doi:10.2140/gt.2008.12.1995

[12] D. Cooper, D. D. Long, and A. W. Reid, Essential closed surfaces in bounded 3-manifolds. J. Amer. Math. Soc. 10 (1997), 553-563. · Zbl 0896.57009 · doi:10.1090/S0894-0347-97-00236-1

[13] D. Cooper and D. D. Long, Some surface subgroups survive surgery. Geom. Topol. 5 (2001), 347-367. · Zbl 1009.57017 · doi:10.2140/gt.2001.5.347

[14] F. Dahmani, Combination of convergence groups. Geom. Topol. 7 (2003), 933-963. · Zbl 1037.20042 · doi:10.2140/gt.2003.7.933

[15] C. Dru\?tu and M. Sapir, Tree-graded spaces and asymptotic cones of groups. Topology 44 (2005), 959-1058. · Zbl 1101.20025 · doi:10.1016/j.top.2005.03.003

[16] B. Farb, Relatively hyperbolic groups. Geom. Funct. Anal. 8 (1998), 810-840. · Zbl 0985.20027 · doi:10.1007/s000390050075

[17] R. Gitik, Ping-pong on negatively curved groups. J. Algebra 217 (1999), 65-72. · Zbl 0936.20019 · doi:10.1006/jabr.1998.7789

[18] M. Gromov, Hyperbolic groups. In Essays in group theory , Math. Sci. Res. Inst. Publ. 8, Springer-Verlag, New York 1987, 75-263. · Zbl 0634.20015

[19] D. Groves and J. F. Manning, Dehn filling in relatively hyperbolic groups. Israel J. Math. 168 (2008), 317-429. · Zbl 1211.20038 · doi:10.1007/s11856-008-1070-6

[20] G. C. Hruska, Relative hyperbolicity and relative quasiconvexity for countable groups. Preprint 2008. · Zbl 1202.20046 · doi:10.2140/agt.2010.10.1807

[21] C. J. Leininger and A. W. Reid, A combination theorem for Veech subgroups of the map- ping class group. Geom. Funct. Anal. 16 (2006), 403-436. · Zbl 1099.57002 · doi:10.1007/s00039-006-0556-9

[22] J. F. Manning and E. Martinez-Pedroza, Separation of relatively quasiconvex subgroups. Preprint 2008. · Zbl 1201.20024 · doi:10.2140/pjm.2010.244.309

[23] B. Maskit, Kleinian groups . Grundlehren Math. Wiss. 287, Springer-Verlag, Berlin 1988. · Zbl 0627.30039

[24] H. A. Masur and Y. N. Minsky, Geometry of the complex of curves I: hyperbolicity. Hyperbolicity. Invent. Math. 138 (1999), 103-149. · Zbl 0941.32012 · doi:10.1007/s002220050343

[25] D. V. Osin, Relatively hyperbolic groups: intrinsic geometry, algebraic properties, and algorithmic problems. Mem. Amer. Math. Soc. 179 (2006), no. 843. · Zbl 1093.20025

[26] D. V. Osin, Peripheral fillings of relatively hyperbolic groups. Invent. Math. 167 (2007), 295-326. · Zbl 1116.20031 · doi:10.1007/s00222-006-0012-3

[27] M. V. Sapir, Some group theory problems. Internat. J. Algebra Comput. 17 (2007), 1189-1214. · Zbl 1172.20032 · doi:10.1142/S0218196707003925

[28] J. Stallings, A finitely presented group whose 3-dimensional integral homology is not finitely generated. Amer. J. Math. 85 (1963), 541-543. · Zbl 0122.27301 · doi:10.2307/2373106

[29] A. Szczepański, Relatively hyperbolic groups. Michigan Math. J. 45 (1998), 611-618. · Zbl 0962.20031 · doi:10.1307/mmj/1030132303

[30] A. Yaman, A topological characterisation of relatively hyperbolic groups. J. Reine Angew. Math. 566 (2004), 41-89. · Zbl 1043.20020 · doi:10.1515/crll.2004.007