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**Supersound many-valued logics and Dedekind-MacNeille completions.** (English)

Zbl 1185.03040

Arch. Math. Logic 48, No. 8, 719-736 (2009).

Authors' abstract: "In [J. Symb. Log. 65, No. 2, 669–682 (2000; Zbl 0971.03025)], *P. Hájek, J. Paris* and *J. Shepherdson* introduce the concept of supersound logic, proving that first-order Gödel logic enjoys this property, whilst first-order Łukasiewicz and product logics do not; in [*P. Hájek* and *J. Shepherdson*, Ann. Pure Appl. Logic 109, No. 1–2, 65–69 (2001; Zbl 1004.03020)] this result is improved showing that, among the logics given by continuous t-norms, Gödel logic is the only one that is supersound. In this paper we will generalize the previous results. Two conditions will be presented: the first one implies the supersoundness and the second one non-supersoundness. To develop these results we will use, between the other machineries, the techniques of completions of MTL-chains developed by C. C. A. Labuschagne and C. J. van Alten. We list some of the main results. The first-order versions of MTL, SMTL, IMTL, WNM, NM, RDP are supersound; the first-order version of an axiomatic extension of BL is supersound if and only if it is  $n$ -potent (i.e. it proves the formula  $\varphi^n \rightarrow \varphi^{n+1}$  for some  $n \in \mathbb{N}^+$ ). Concerning the negative results, we have that the first-order versions of IIMTL, WCMTL and of each non- $n$ -potent axiomatic extension of BL are not supersound."

Reviewer: **Daniele Mundici (Firenze)**

**MSC:**

**03B50** Many-valued logic

**06B23** Complete lattices, completions

Cited in 4 Documents

**Keywords:**

Gödel logic; many-valued logic; completion of MTL-chains; axiomatic extension

**Full Text:** DOI

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