Farhat, C.; **Kalashnikova, I.**; **Tezaur, R.**

A higher-order discontinuous enrichment method for the solution of high Péclet advection-diffusion problems on unstructured meshes. (English) Zbl 1183.76805

Int. J. Numer. Methods Eng. 81, No. 5, 604-636 (2010).

Summary: A higher-order discontinuous enrichment method (DEM) with Lagrange multipliers is proposed for the efficient finite element solution on unstructured meshes of the advection-diffusion equation in the high Péclet number regime. Following the basic DEM methodology, the usual Galerkin polynomial approximation is enriched with free-space solutions of the governing homogeneous partial differential equation (PDE). In this case, these are exponential functions that exhibit a steep gradient in a specific flow direction. Exponential Lagrange multipliers are introduced at the element interfaces to weakly enforce the continuity of the solution. The construction of several higher-order DEM elements fitting this paradigm is discussed in detail. Numerical tests performed for several two-dimensional benchmark problems demonstrate their computational superiority over stabilized Galerkin counterparts, especially for high Péclet numbers.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
74R99 Fracture and damage
80A20 Heat and mass transfer, heat flow (MSC2010)

Keywords:

advection-diffusion; discontinuous Galerkin method; discontinuous enrichment method; high Péclet number; Lagrange multipliers; high-order

Full Text: DOI

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