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Instabilities in free-surface Hartmann flow at low magnetic Prandtl numbers. (English)

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Summary: We study the linear stability of the flow of a viscous electrically conducting capillary fluid on a planar fixed plate in the presence of gravity and a uniform magnetic field, assuming that the plate is either a perfect electrical insulator or a perfect conductor. We first confirm that the Squire transformation for magnetohydrodynamics is compatible with the stress and insulating boundary conditions at the free surface but argue that unless the flow is driven at fixed Galilei and capillary numbers, respectively parameterizing gravity and surface tension, the critical mode is not necessarily two-dimensional. We then investigate numerically how a flow-normal magnetic field and the associated Hartmann steady state affect the soft and hard instability modes of free-surface flow, working in the low-magnetic-Prandtl-number regime of conducting laboratory fluids ($Pm \leq 10^{-4}$). Because it is a critical-layer instability (moderately modified by the presence of the free surface), the hard mode exhibits similar behaviour as the even unstable mode in channel Hartmann flow, in terms of both the weak influence of Pm on its neutral-stability curve and the dependence of its critical Reynolds number Re_c on the Hartmann number Ha. In contrast, the structure of the soft mode's growth-rate contours in the (Re, α) plane, where α is the wavenumber, differs markedly between problems with small, but non-zero, Pm and their counterparts in the inductionless limit, $Pm \searrow 0$. As derived from large-wavelength approximations and confirmed numerically, the soft mode's critical Reynolds number grows exponentially with Ha in inductionless problems. However, when Pm is non-zero the Lorentz force originating from the steady-state current leads to a modification of $Re_c(Ha)$ to either a sub-linearly increasing or a decreasing function of Ha, respectively for problems with insulating or perfectly conducting walls. In insulating-wall problems we also observe pairs of counter-propagating Alfvén waves, the upstream-propagating wave undergoing an instability driven by energy transferred from the steady-state shear to both of the velocity and magnetic degrees of freedom. Movies are available with the online version of the paper.

MSC:

76E25 Stability and instability of magnetohydrodynamic and electrohydrodynamic flows

76W05 Magnetohydrodynamics and electrohydrodynamics

Full Text: DOI

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