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Pure equilibrium tetrahedral finite elements for global error estimation by dual analysis.

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Summary: This study presents a general procedure of creating pure equilibrium tetrahedral finite elements for use under the elastostatic hypothesis. These pure equilibrium elements are of the Fraeijs de Veubeke type and the degree of the polynomial approximation functions of their internal stress field is the parameter generating this new elements family. The spurious kinematic modes (SKM), inherent in the equilibrium approach, are eliminated at the element level by converting each tetrahedron into a super-element defined as an assembly of four tetrahedral primitive elements. A mathematical discussion on the number of SKM of the primitive elements as well as their elimination by the super-element technique has been carried out. The development of first and second degree elements is presented here in detail and their efficiency in the frame of global error estimation by dual analysis is emphasized by two numerical applications. The main attribute of the error estimation by dual analysis is that it provides an upper bound on the global discretization error.

MSC:

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