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An improved subspace selection algorithm for meshless collocation methods. (English)

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Summary: Choosing data points is a common problem for researchers who employ various meshless methods for solving partial differential equations. On the one hand, high accuracy is always desired; on the other, ill-conditioning problems of the resultant matrices, which may lead to unstable algorithms, prevent some researchers from using meshless methods. For example, the optimal placements of source points in the method of fundamental solutions or of the centers in the radial basis functions method are always unclear. Intuitively, such optimal locations will depend on many factors: the partial differential equations, the domain, the trial basis used (i.e. the employed method itself 1pt), the computational precisions, some user-defined parameters, and so on. Such complexity makes the hope of having an optimal centers placement unpromising. In this paper, we provide a data-dependent algorithm that adaptively selects centers based on all the other variables.

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65N35 Spectral, collocation and related methods for boundary value problems involving PDEs

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radial basis function; Kansa's method; collocation; adaptive greedy algorithm

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