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On the generalized Hyers-Ulam-Rassias stability of higher ring derivations. (English)

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Let \mathcal{A} , \mathcal{B} be real or complex algebras. A sequence $H = \{h_0, h_1, \dots\}$ of additive operators from \mathcal{A} to \mathcal{B} is called a *higher ring derivation* if

$$h_n(zw) = \sum_{i=0}^n h_i(z)h_{n-i}(w), \quad z, w \in \mathcal{A}, n = 0, 1, \dots$$

A sequence $F = \{f_0, f_1, \dots\}$ of operators from \mathcal{A} to \mathcal{B} is called a *higher derivation* if

$$f_n(x + y + zw) = f_n(x) + f_n(y) + \sum_{i=0}^n f_i(z)f_{n-i}(w), \quad x, y, z, w \in \mathcal{A}, n = 0, 1, \dots$$

The main goal of the paper is to consider approximate higher derivations and the problem of the stability of higher ring derivations. It is shown, in particular, that if a sequence $F = \{f_0, f_1, \dots\}$ satisfies, with some given control mappings $\varphi_n : \mathcal{A}^4 \rightarrow [0, \infty)$,

$$\|f_n(x + y + zw) - f_n(x) - f_n(y) - \sum_{i=0}^n f_i(z)f_{n-i}(w)\| \leq \varphi_n(x, y, z, w)$$

for all $x, y, z, w \in \mathcal{A}$ and $n = 0, 1, \dots$, then there exists a unique higher ring derivation $H = \{h_0, h_1, \dots\}$ such that h_n is somehow *close* to f_n for each n .

Several corollaries are obtained for particular control mappings φ_n and under some additional assumptions upon \mathcal{A} .

The results refer in particular to *D. G. Bourgin* [Duke Math. J. 16, 385–397 (1949; Zbl 0033.37702)], *R. Badora* [Math. Inequal. Appl. 9, No. 1, 167–173 (2006; Zbl 1093.39024)], *T. Miura*, *G. Hirasawa* and *S.-E. Takahasi* [J. Math. Anal. Appl. 319, No. 2, 552–530 (2006; Zbl 1104.39025)].

Reviewer: **Jacek Chmieliński** (Kraków)

MSC:

39B82 Stability, separation, extension, and related topics for functional equations

39B52 Functional equations for functions with more general domains and/or ranges

Keywords:

higher ring derivation; Hyers-Ulam-Rassias stability; Banach algebra

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