Spada, Luca

μMV-algebras: An approach to fixed points in Łukasiewicz logic. (English) Zbl 1183.06006

Fuzzy Sets Syst. 159, No. 10, 1260-1267 (2008).

Chang introduced MV-algebras to give a purely algebraic proof of the completeness of the Łukasiewicz axioms. In his paper [J. Funct. Anal. 65, 15–63 (1986; Zbl 0597.46059)], the present reviewer established a categorical equivalence between MV-algebras and unital lattice-ordered abelian groups. Divisible MV-algebras are the correspondents of divisible unital lattice-ordered abelian groups. By further equipping a divisible MV-algebra with the well-known Δ-operator, one has divisible MV_Δ-algebras. The author proves that by expanding MV-algebras to structures allowing minimal and maximal fixed points, one obtains a term-equivalent variant of divisible MV_Δ-algebras. He then derives various kinds of results on these algebras, such as subdirect representation, completeness, amalgamation and a representation of free algebras. For background on MV-algebras see the monograph [*R. L. O. Cignoli, I. M. L. D'Ottaviano* and *D. Mundici*, Algebraic foundations of many-valued reasoning. Dordrecht: Kluwer Academic Publishers (2000; Zbl 0937.06009)].

Reviewer: Daniele Mundici (Firenze)

MSC:

06D35 MV-algebras
03B50 Many-valued logic
03G25 Other algebras related to logic

Keywords:

MV-algebra; lattice-ordered group; many-valued logic; divisible MV-algebra; Δ-operator; fixed point

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