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Combined maximality principles up to large cardinals. (English) Zbl 1182.03078

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Let $\varphi(a)$ be a formula and let Γ be a definable (possibly with parameters) class of forcings. Then $\varphi(a)$ is said to be Γ -forceable if there is a $\mathbb{P} \in \Gamma$ such that $\mathbb{P} \Vdash \varphi(\check{a})$. Further, $\varphi(a)$ is Γ -necessary if for every $\mathbb{P} \in \Gamma$, $\mathbb{P} \Vdash \varphi(\check{a})$. It is Γ -forceably necessary if there is a $\mathbb{P} \in \Gamma$ such that $\mathbb{P} \Vdash \forall \mathbb{Q} (\mathbb{Q} \in \Gamma \Rightarrow \mathbb{Q} \Vdash \varphi(\check{a}))$. The Maximality Principle for forcings in Γ with parameters in P , denoted by $\text{MP}_\Gamma(P)$, is the formula scheme that says that every formula with parameters in P that is Γ -forceably necessary is true.

In the paper under review, the author investigates the possibility of combining Maximality Principles where the forcings involved are $< \kappa$ -closed or $< \kappa$ -directed-closed and $P \subseteq H_{\kappa^+}$, for κ a regular cardinal. He begins by constructing models where the directed closed Maximality Principle holds below a large cardinal and shows that certain combinations have high consistency strength. He follows with the construction of models where the directed closed Maximality Principle holds up to and *including* a large cardinal. Finally, the author considers combinations of Maximality Principles up to and including cardinals that are partially supercompact or Woodinized supercompact.

Reviewer: [J. M. Plotkin \(East Lansing\)](#)

MSC:

[03E35](#) Consistency and independence results

[03E55](#) Large cardinals

Cited in **8** Documents

Keywords:

[maximality principle](#); [large cardinal](#)

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References:

- [1] Closed maximality principles: Implications, separations and combinations 73 pp 276– (2008)
- [2] Handbook of set theory
- [3] DOI: 10.1002/1521-3870(200111)47:4<563::AID-MALQ563>3.0.CO;2-# · [Zbl 0992.03064](#) · doi:10.1002/1521-3870(200111)47:4<563::AID-MALQ563>3.0.CO;2-
- [4] Kobe Journal of Mathematics 16 pp 119– (1999)
- [5] Inner models and large cardinals (2002)
- [6] Reflection principles for the continuum (2001)
- [7] DOI: 10.4064/fm180-3-4 · [Zbl 1066.03052](#) · doi:10.4064/fm180-3-4
- [8] The higher infinite (2003)
- [9] Independence of the axiom of dependent choices from the countable axiom of choice 31 pp 294– (1966)
- [10] Stacking mice 74 pp 315– (2009)
- [11] Set theory (2003)
- [12] A simple maximality principle 68 pp 527– (2003)
- [13] Annals of Mathematical Logic 7 pp 327– (1974)

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