Wei, Long

Concentrating phenomena in some elliptic Neumann problem: asymptotic behavior of solutions. (English) Zbl 1181.35106

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For a bounded smooth domain $\Omega \subseteq \mathbb{R}^2$, smooth, positive $a\colon \overline{\Omega} \to \mathbb{R}$, and small positive ε consider

$$\begin{cases} -\operatorname{div}(a(x)\nabla u) + a(x)u = 0, & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \varepsilon \mathrm{e}^u, & \text{in } \partial\Omega. \end{cases} \tag{P_ε}$$

The problem is a generalization of the one considered in [*J. Dávila, M. del Pino, M. Musso*, J. Funct. Anal. 227, No. 2, 430–490 (2005; Zbl 1207.35158)], where $a \equiv 1$. Suppose that $\varepsilon_n \to 0$ and that u_n is a solution of (P_{ε_n}), for each $n \in \mathbb{N}$. If $\varepsilon_n \int_{\partial\Omega} \mathrm{e}^{u_n}\,\mathrm{d}x$ remains bounded as $n \to \infty$, it is shown that then, after passing to a subsequence, either u_n remains bounded in $L^\infty(\Omega)$ or u_n blows up in a finite number of points on $\partial\Omega$ that are critical points of $a|_{\partial\Omega}$.

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MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
35J20 Variational methods for second-order elliptic equations
35J25 Boundary value problems for second-order elliptic equations
35J67 Boundary values of solutions to elliptic equations and elliptic systems

Keywords:

nonlinear boundary value problem; exponential Neumann nonlinearity; boundary concentration

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