Cignoli, Roberto; Esteva, Francesc

Commutative integral bounded residuated lattices with an added involution. (English)

Zbl 1181.03061

Ann. Pure Appl. Logic 161, No. 2, 150-160 (2009).

A symmetric residuated lattice is an algebra $A = (A, \vee, \wedge, *, \to, \sim, 1, 0)$ such that $(A, \vee, \wedge, *, \to, 1, 0)$ is a commutative integral bounded residuated lattice and the equations $\sim\sim x = x$ and $\sim (x \vee y) = \sim x \wedge \sim y$ are satisfied.

The aim of this paper is to investigate the properties of the unary operation ε defined by $\varepsilon x = \sim x \to 0$.

The authors give necessary and sufficient conditions for ε to be an interior operator. Since these conditions are rather restrictive, they consider when an iteration of ε is an interior operator. In particular they consider the chain of varieties of symmetric residuated lattices such that the n-fold iteration of ε is a Boolean interior operator. They show that these varieties are semisimple. For $n = 1$, the variety of symmetric Stonean residuated lattices is obtained. Also, the authors characterize the subvarieties admitting representations as subdirect products of chains.

Reviewer: Dana Piciu (Craiova)

MSC:

03G25 Other algebras related to logic

03B47 Substructural logics (including relevance, entailment, linear logic, Lambek calculus, BCK and BCI logics)

03B52 Fuzzy logic; logic of vagueness

06F05 Ordered semigroups and monoids

Keywords:

residuated lattices; pseudo-complemented residuated lattices; Stonean residuated lattices; order-reversing involutions; interior operators

Full Text: DOI

References:

[1] Baaz, M., Infinite-valued Gödel logics with 0-1 projections and relativizations, (), 23-33 · Zbl 0862.03015

[2] Balbes, R.; Dwinger, P., Distributive lattices, (1974), University of Missouri Press Columbia, Miss · Zbl 0321.06012

[3] Boixader, D., Some properties concerning the quasi-inverse of a t-norm, Mathware soft comput., 5, 5-12, (1998) · Zbl 0932.03065

[4] Burris, S.; Sankappanavar, H.P., A course in universal algebra, (1981), Springer-Verlag New York · Zbl 0478.08001

[5] Cignoli, R., Free algebras in varieties of Stonean residuated lattices, Soft comput., 12, 315-320, (2008) · Zbl 1142.06003

[6] Cignoli, R.; de Gallego, M.S., The lattice structure of some łukasiewicz algebras, Algebra universalis, 13, 315-328, (1981) · Zbl 0495.03045

[7] Cignoli, R.; Esteva, F.; Godo, L.; Torrens, A., Basic logic is the logic of continuous t-norms and their residua, Soft comput., 4, 106-112, (2000)

[8] Cignoli, R.; Esteva, F.; Godo, L.; Montagna, F., On a class of left-continuous t-norms, Fuzzy sets and systems, 131, 283-296, (2002) · Zbl 1012.03032

[9] Cignoli, R.; Torrens, A., Standard completeness of Hájek basic logic and decompositions of BL-chains, Soft comput., 9, 862-868, (2005) · Zbl 1094.03013

[10] Cintula, P.; Klement, E.P.; Mesiar, R.; Navara, M., Residuated logics based on strict triangular norms with an involutive negation, Math. log. Q., 52, 269-282, (2006) · Zbl 1165.03326

[11] Esteva, F.; Godo, L., Monoidal t-norm based logic: towards a logic for left-continuous t-norms, Fuzzy sets and systems, 124, 271-288, (2001) · Zbl 0994.03017

[12] Esteva, F.; Godo, L.; Hájek, P.; Navara, M., Residuated fuzzy logics with an involutive negation, Arch. math. logic, 39, 103-124, (2000) · Zbl 0965.03035

[13] Esteva, F.; Godo, L.; García-Cerdeña, A., On the hierarchy of t-norm based residuated fuzzy logics, (), 251-272 · Zbl

1038.03026

[14] Flaminio, T.; Marchioni, E., T-norm based logic with an independent involutive negation, Fuzzy sets and systems, 157, 3125-3144, (2006) · Zbl 1114.03015

[15] Freytes, H., Injectives in residuated algebras, Algebra universalis, 51, 373-393, (2004) · Zbl 1079.06014

[16] Galatos, N.; Jipsen, P.; Kowalski, T.; Ono, H., ()

[17] Grätzer, G., General lattice theory, (1978), Birkhäuser Verlag Basel, Stuttgar · Zbl 0385.06015

[18] Hájek, P., Methamathematics of fuzzy logic, (1998), Kluwer Dordrecht, Boston, London

[19] Hájek, P., Basic fuzzy logic and BL-algebras, Soft comput., 2, 124-128, (1998)

[20] Höle, U., Commutative, residuated l-monoids, (), 53-106 · Zbl 0838.06012

[21] Jenei, S.; Montagna, F., A proof of standard completeness for esteva and godo's logic MTL, Studia. logica., 70, 183-192, (2002) · Zbl 0997.03027

[22] T. Kowalski, H. Ono, Residuated lattices: An algebraic glimpse at logics without contraction, Preliminary report

[23] Moisil, G., Logique modale, Disquisit. math. phys., 2, 3-98, (1942), Reproduced in [24,~pp. 341-431] · Zbl 0063.04064

[24] Moisil, G., Essais sur LES logiques non chrysippiennes, (1972), Editions de l'Academie de la Republique Socialiste de Roumanie Bucharest · Zbl 0241.02006

[25] Monteiro, A.A., Sur LES algèbres de Heyting symmétriques, Port. math., 39, 1-237, (1980)

[26] Sankappanavar, H.P., Heyting algebras with a dual lattice endomorphism, Z. math. logik grundlag. math., 33, 565-573, (1987) · Zbl 0633.06005