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Metric identities and the discontinuous spectral element method on curvilinear meshes. (English) Zbl 1178.76269

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Summary: We study how to approximate the metric terms that arise in the discontinuous spectral element (DSEM) approximation of hyperbolic systems of conservation laws when the element boundaries are curved. We first show that the metric terms can be written in three forms: the usual cross product and two curl forms. The first curl form is identical to the "conservative" form presented by *P. D. Thomas* and *C. K. Lombard* [AIAA J. 17, 1030–1037 (1979; Zbl 0436.76025)]. The second is a coordinate invariant form. We prove that in two space dimensions, the typical approximation of the cross product form does satisfy a discrete set of metric identities if the boundaries are isoparametric and the quadrature is sufficiently precise. We show that in three dimensions, this cross product form does not satisfy the metric identities, except in exceptional circumstances. Finally, we present approximations of the curl forms of the metric terms that satisfy the discrete metric identities. Two examples are presented to illustrate how the evaluation of the metric terms affects the satisfaction of the discrete metric identities, one in two space dimensions and the other in three.

MSC:

76M22 Spectral methods applied to problems in fluid mechanics
78M25 Numerical methods in optics (MSC2010)

Cited in **58** Documents

Full Text: DOI

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