Taube, Arne; Dumbser, Michael; Balsara, Dinshaw S.; Munz, Claus-Dieter

Arbitrary high-order discontinuous Galerkin schemes for the magnetohydrodynamic equations. (English) Zbl 1176.76075

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Summary: We propose a discontinuous Galerkin scheme with arbitrary order of accuracy in space and time for magnetohydrodynamic equations. It is based on the Arbitrary order using DERivatives (ADER) methodology: the high order time approximation is obtained by a Taylor expansion in time. In this expansion, all the time derivatives are replaced by space derivatives via the Cauchy-Kovalevskaya procedure. We propose an efficient algorithm of the Cauchy-Kovalevskaya procedure in the case of the three-dimensional magneto-hydrodynamic (MHD) equations. Parallel to the time derivatives of the conservative variables, the time derivatives of the fluxes are calculated. This enables analytic time integration of the volume integral as well as the surface integral of the fluxes through the grid cell interfaces which occur in discrete equations. At the cell interfaces, the fluxes and all their derivatives may jump. Following the finite volume ADER approach the break up of all these jumps into the different waves are taken into account to get proper values of the fluxes at the grid cell interfaces. The approach under considerations is directly based on the expansion of the flux in time in which the leading order term may be any numerical flux calculation for the MHD-equation. Numerical convergence results for these equations up to 7th order of accuracy in space and time are presented.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
76W05 Magnetohydrodynamics and electrohydrodynamics

Keywords:

discontinuous Galerkin finite elements; ADER approach; flux expansion; generalized Riemann problem; Cauchy-Kovalevskaya procedure; magnetohydrodynamics

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