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Quantitative uniform distribution of points of small height on the projective line. (Equidistribution quantitative des points de petite hauteur sur la droite projective.) (English)

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Summary: We introduce a new class of adelic heights on the projective line. We estimate their essential minimum and prove a result of uniform distribution (at every place) for points of small height with estimates on the speed of convergence. To each rational function R in one variable and defined over a number field K, is associated a normalized height on the algebraic closure of K. We show that these dynamically defined heights are adelic in our sense, and deduce from this uniform distribution results for preimages of points under R at every place of K. Our approach follows that of Bilu, and relies on potential theory in the complex plane, as well as in the Berkovich space associated to the projective line over $\mathbb{C}_p$, for each prime p.

MSC:

11G50 Heights
37F10 Dynamics of complex polynomials, rational maps, entire and meromorphic functions; Fatou and Julia sets

Full Text: DOI

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