Babadjian, Jean-François; Barchiesi, Marco
A variational approach to the local character of $G$-closure: the convex case. (English)
Zbl 1173.35012

The problem of the determination of the $G$-closure is here raised within the nonlinear elasticity framework. The authors consider $N$ energy densities $W^{(i)}$, $i = 1, \ldots, N$, which are continuous functions belonging to the set $F(\alpha, p)$, that is satisfying a $p$-growth and a $p$-coercivity condition, with $p > 1$. A composite material is obtained as a mixture of these different basic materials considering the energy density $W_\theta = \sum_i \theta_i W^{(i)}$, for some functions $\theta_i \in L^\infty(\Omega; [0, 1])$, satisfying $\sum \theta_i = 1$, a.e. in $\Omega$.

One main result of the paper characterizes the $G$-closure. Every effective energy density in this $G$-closure can be locally recovered through a $\Gamma$-convergence process as the pointwise limit of the sequence of periodic homogenized energy densities with prescribed volume functions. This is more precisely obtained taking a sequence $(\chi_k)$ which satisfies $\sum_k \chi_k^{(i)} = 1$, a.e. in $\Omega$, where $\chi_k(y) = \chi(\langle ky \rangle)$ and which converges to $\theta$ in the weak* topology of $L^\infty(\Omega; [0, 1]^N)$. The authors first claim that they can assume the quasiconvexity of the energy densities. The proof of this result is based on the properties of the $\Gamma$-convergence (for the weak topology of $W^{1,p}$) and on classical functional analysis tools. When removing the convexity condition, but assuming that the zero-level set of the effective energy density can be associated to Young measures, the authors prove some similar characterization of the $G$-closure. The authors also discuss the behaviour of cell integrands. The paper ends with some counter-examples to the results or properties exposed in the results obtained throughout the paper. One of these counter-examples deals with the possibility to obtain similar results when considering a two-scale convergence method.

Reviewer: Alain Brillard (Riedisheim)

MSC:
35B27 Homogenization in context of PDEs; PDEs in media with periodic structure
35B40 Asymptotic behavior of solutions to PDEs
74B20 Nonlinear elasticity
74E30 Composite and mixture properties
74Q05 Homogenization in equilibrium problems of solid mechanics

Keywords:
$\Gamma$-convergence; quasiconvexity; polyconvexity; Young measure; two-scale convergence; cell integrands; counter-examples

Full Text: DOI arXiv EuDML

References: