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The variational approach to fracture mechanics. A practical application to the French Panthéon in Paris. (English) [Zbl 1166.74029](#)

J. Elasticity 95, No. 1-2, 1-30 (2009).

G. A. Francfort and *J.-J. Marigo* [*J. Mech. Phys. Solids* 46, No. 8, 1319–1342 (1998; [Zbl 0966.74060](#))] have proposed a novel approach to fracture mechanics based upon the global minimization of a Griffith-like functional, composed of a bulk and a surface energy terms. In 2000, the same authors, together with *B. Bourdin*, introduced in [*J. Mech. Phys. Solids* 48, No. 4, 797–826 (2000; [Zbl 0995.74057](#))] a variational approximation (in the sense of Γ -convergence) of such functional, essentially for computational purposes. Here, the authors utilize this new variational approach to show how it might be altered to incorporate the idea of less brittle, "deviatoric-type fracture", and apply this theory to materials such as confined stone. The authors modify the original formulation of Francfort and Marigo, in particular the approximation of Bourdin, Francfort and Marigo, to allow for discontinuities in the deviatoric part of the strain. The model is applied to study the masonry of the French Panthéon. Numerical experiments are also provided. The modified formulation allows one to reproduce fracture paths surprisingly similar to that observed in situ, to sort out the possible causes of damage, and to confirm, with a quantitative analysis, the main structural deficiencies in the French monument.

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MSC:

- [74R10](#) Brittle fracture
- [74R05](#) Brittle damage
- [74G65](#) Energy minimization in equilibrium problems in solid mechanics
- [74S05](#) Finite element methods applied to problems in solid mechanics

Cited in **34** Documents

Keywords:

variational calculus; free-discontinuity problems; quasi-brittle materials; finite elements; Stone-Ashlar masonry

Full Text: [DOI](#)

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