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On the number of interior peaks of solutions to a non-autonomous singularly perturbed Neumann problem. (English) [Zbl 1162.35009](#)

Proc. R. Soc. Edinb., Sect. A, Math. 139, No. 2, 427-448 (2009).

Let $N \geq 2$ and suppose that $\Omega \subseteq \mathbb{R}^N$ is a bounded smooth domain, $a \in C^\infty(\overline{\Omega})$, and $1 < p < 2^* - 1$, where 2^* is the usual critical Sobolev exponent that corresponds to this problem. It is never stated but implicitly assumed that a is positive. For a small positive parameter ε consider the elliptic problem

$$\begin{cases} -\varepsilon^2 \Delta u + u = a(x)u^p & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega. \end{cases} \quad (1)$$

Here $\partial/\partial\nu$ denotes the partial derivative in the normal direction on $\partial\Omega$.

If $Q_0 \in \Omega$ is a strict local minimum of a then it is shown that there are $\varepsilon_0 > 0$ and $K_0(\varepsilon) > 0$, $K_0(\varepsilon) \rightarrow \infty$ as $\varepsilon \rightarrow 0$, such that for $\varepsilon \in (0, \varepsilon_0]$ and $K \leq K_0(\varepsilon)$ there is a multipeak solution of (1) with exactly K local maximum points. These maxima tend to Q_0 as $\varepsilon \rightarrow 0$.

In the result of [*F.-H. Lin, W.-M. Ni and J.-C. Wei*, Commun. Pure Appl. Math. 60, No. 2, 252-281 (2007; [Zbl 1170.35424](#))], a similar statement is made for the case $a \equiv 1$, with asymptotics $K_0(\varepsilon) \sim \varepsilon^{-N} |\log \varepsilon|^{-N}$. In contrast, in the paper under review the asymptotics are $K_0(\varepsilon) \sim \varepsilon^{-M} |\log \varepsilon|^{-N}$ for some $M \in (0, 1)$ that depends on the order of the zero of ∇a in Q_0 .

Reviewer: [Nils Ackermann \(México\)](#)

MSC:

- [35B25](#) Singular perturbations in context of PDEs
- [35J20](#) Variational methods for second-order elliptic equations
- [35J25](#) Boundary value problems for second-order elliptic equations

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Keywords:

[multipeak solution](#); [interior peaks](#); [asymptotic estimate](#); [number of peaks](#); [Neumann problem](#)

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