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A theory of semilinear operator equations under nonresonance conditions. (English)

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Given a separable Hilbert space H with inner product $\langle\cdot,\cdot\rangle_H$ and a selfadjoint positive operator $A : D(A) \to H$ in H with compact resolvent, the energy space H_A of A is defined as $H_A := A^{-1/2}(H)$, with the inner product $\langle u, v\rangle_{H_A} := \langle A^{1/2}u, A^{1/2}v\rangle_H$. Then the nonlinear operator equation

$$\begin{cases} Au = cu + F(u), \\ u \in H_A, \end{cases} \tag{1}$$

is considered in the non-resonant case $c \notin \sigma(A)$. Here, one looks for weak solutions of (1), i.e., elements $u \in H_A$ such that $\langle u, v\rangle_{H_A} = \langle cu + F(u), v\rangle_H$ for all $v \in H_A$.

Using Banach's fixed point theorem and Leray–Schauder theory, a survey of various well-known existence theorems is given for (1). The conditions on F amount to at most linear growth in u and v, and in some instances include (partial) global Lipschitz conditions on F. The growth and Lipschitz constants appearing here are of the order $1/\|(A - c)^{-1}\|$.

Finally, it is shown how these theorems can be applied to semilinear elliptic partial differential equations.

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MSC:

47J05	Equations involving nonlinear operators (general)
35J65	Nonlinear boundary value problems for linear elliptic equations
47J25	Iterative procedures involving nonlinear operators
47N20	Applications of operator theory to differential and integral equations
35R20	Operator partial differential equations (= PDEs on finite-dimensional spaces for abstract space valued functions)
35D05	Existence of generalized solutions of PDE (MSC2000)
35J60	Nonlinear elliptic equations
47H10	Fixed-point theorems

Keywords:
nonlinear operator equation; fixed point; Leray-Schauder theory; semilinear elliptic equation