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A criterion for existence of a positive solution of a nonlinear elliptic system. (English) Zbl 1161.35012

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The author treats an elliptic system with gradient structure, governed by the pair (p,q)-Laplacian, posed on a smoothly bounded domain Ω in $\mathbb{R}^N$:

$$\begin{cases} -\Delta_{p_1} u = \frac{\partial H}{\partial u}(x;u,v), & x \in \Omega, \\ -\Delta_{p_2} v = \frac{\partial H}{\partial v}(x;u,v), & x \in \Omega, \\ u(x) = v(x) = 0, & x \in \partial\Omega. \end{cases} \tag{1}$$

Here $1 < p_i < N$ and $\Delta_p := \operatorname{div}(|\nabla|^{p-2}\nabla)$ denotes the p-Laplacian, as usual. The potential function H is given by

$$H(x;s,t) := \int_0^s h_1(x;r)\,dr + \int_0^t h_2(x;r)\,dr + \left(\int_0^s g_1(x;r)\,dr\right)\left(\int_0^t g_2(x;r)\,dr\right),$$

where h_i and g_i are positive Caratheodory functions, $i = 1, 2$. It is assumed that the functions h_i are asymptotically homogeneous of orders $p_i - 1$ in the second argument r, for r near 0 and ∞, and that the functions g_i are asymptotically homogeneous near $r = 0$, of orders less than $p_i - 1$. Some monotonicity conditions are imposed on h_i and g_i, and concavity conditions on g_i.

The result states a sharp criterion for the existence of a unique positive solution to (1). It is formulated as a sign condition on the lowest eigenvalues of three related nonlinear eigenvalue problems. The proof is variational and employs the gradient structure of the problem.

Reviewer: Nils Ackermann (México)

MSC:

35J55 Systems of elliptic equations, boundary value problems (MSC2000)
35J50 Variational methods for elliptic systems

Keywords:

elliptic system; p-Laplacian; positive solution; nonlinear eigenvalue

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