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A volumetric integral radial basis function method for time-dependent partial differential equations. I. Formulation. (English) Zbl 1159.76363

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Summary: A strictly conservative volume integral formulation of the time dependent conservation equations in terms of meshless radial basis functions (RBFs) is presented. Rotational and translational transformations are considered that simplify the partial differential equations (PDEs) to be solved. As a result, the solutions that are represented at a finite sample of knots, $x \in \Omega\partial\Omega \subset \mathbb{R}^d$ are permitted to move as the system of equations evolves in time. Knots are inserted, deleted, or rearranged in such a manner to conserve the extensive physical quantities of mass, momentum components, and total energy.

Our study consists of the following parts:

(A) Local rotational and Galilean translational transformations can be obtained to reduce the conservation equations into steady-state forms for the inviscid Euler equations or Navier — Stokes equations.

(B) The entire set of PDEs are transformed into the method of lines approach yielding a set of coupled ordinary differential equations whose homogeneous solution is exact in time.

(C) The spatial components are approximated by expansions of meshless RBFs; each individual RBF is volumetrically integrated at one of the sampling knots x_i, yielding a collocation formulation of the method of lines structure of the ODEs

.(D) Because the volume integrated RBFs increase more rapidly away from the data center than the commonly used RBFs, we use a higher order preconditioner to counter-act the ill-conditioning problem. Domain decomposition is used over each piecewise continuous subdomain.

MSC:

76M25 Other numerical methods (fluid mechanics) (MSC2010)
76N15 Gas dynamics (general theory)

Keywords:

Galilean transformations; momentum component transformations; shock waves; volumetric formulation

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HE-E1GODF

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