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Derived categories of toric varieties. (English) Zbl 1159.14026

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In this paper, the author investigates the structure of the derived category of a toric variety. If X is a projective toric variety with at most quotient singularities, and B is an invariant \mathbb{Q} -divisor with coefficients of the form $(r-1)/r$ with r a nonnegative integer, then one can consider the smooth Deligne-Mumford stack \mathcal{X} associated to the pair (X, B) , as in Y. Kawamata [J. Math. Sci. Univ. Tokyo 12, 211–231 (2005; Zbl 1095.14014)]. The author shows that the derived category $D^b(\mathcal{X})$ has a complete exceptional collection consisting of sheaves.

In order to prove the result, the author considers a projective space, which is known to have a complete exceptional collection. Using the toric minimal program, he then constructs a complete exceptional collection on any toric variety with at most quotient singularities. Indeed, by a covering trick he proceeds from projective spaces to log Fano varieties. It is now enough to work out Mori fiber spaces: the presence of multiple fibres shows that boundary cases have to be taken into account, and this introduces the branch divisor B into the picture. Even if a Mori fiber space can have singular fibers, the associated morphism of stacks is smooth. Then a careful study of “stacky” sheaves gives the required collection (which consists indeed of sheaves) on any toric variety with at most quotient singularities.

Reviewer: [Marcello Bernardara \(Bonn\)](#)

MSC:

14M25 Toric varieties, Newton polyhedra, Okounkov bodies
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)
18E30 Derived categories, triangulated categories (MSC2010)

Cited in **3** Reviews
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Keywords:

toric varieties; derived categories; exceptional collection; toric minimal model program

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