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A well-balanced approach for flows over mobile-bed with high sediment-transport. (English)

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Summary: We deal with the numerical computation of one-dimensional, unsteady, free-surface flows over mobile-bed. We focus on flows characterized by high concentration of sediments and strong interaction between flow and bottom dynamics, as in hyper-concentrated- and debris-flows. These features are fully considered in the adopted system of equations. Challenging in its numerical approximation is the preservation of the coupling and the treatment of a non-conservative flux in the momentum equation. In order to devise a new Godunov-type approach, we analyzed in detail the Riemann problem associated with the mobile-bed phenomena and the peculiar features of its wave relations. The scheme we developed is based on two supports: well-balanced treatment of the variable updating at the new time-level and flux evaluation by three-wave approximations of the intercell Riemann-problem that, without any split, embody the effect of the non-conservative term. The properties of the new numerical strategy (named AWB) are assessed by comparison with exact solutions of Riemann problems, built by handling an inverse technique. Finally, AWB has been applied to cases of practical interest, where wave interaction and friction effects makes the flow more complex. The obtained results point out that the new method is able to predict faithfully the overall behaviour of the solution and of any type of waves. The use of AWB, in this one-dimensional frame, is therefore fostered in representing rapid transients in river/torrent flows with movable bed.

MSC:

86-08 Computational methods for problems pertaining to geophysics
76B07 Free-surface potential flows for incompressible inviscid fluids
76M20 Finite difference methods applied to problems in fluid mechanics
86A05 Hydrology, hydrography, oceanography

Keywords:

well-balanced scheme; Riemann solvers; non-conservative term; mobile-bed models; sediment transport

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