Megrelishvili, Michael
Reflexively representable but not Hilbert representable compact flows and semitopological semigroups. (English) [Zbl 1158.37008]

Actions \((s, x) \mapsto sx\) of semitopological semigroups on compact metric spaces which are separately continuous in \(s\) and in \(x\) are considered. The action is said to be reflexively representable if there exists a proper representation of the action on a reflexive Banach space \(V\). If one can choose \(V\) to a Hilbert space then the action is said to be Hilbert representable. A function \(f\) is said to be weakly almost periodic if its orbit is relatively weakly compact. The Fourier-Stieltjes algebra of a topological group is the set of all matrix coefficients of Hilbert representations. One calls a topological group an Eberlein group if the Fourier-Stieltjes algebra is dense in the set of all weakly almost periodic functions.

Let \(G\) be a separable topological group such that every reflexively representable transitive compact metric \(G\)-flow is Hilbert representable. It is proved that \(G\) is an Eberlein group. Using a result from W. Rudin [Duke Math. J. 26, 215-220 (1959; Zbl 0085.32502)] stating that \(\mathbb{Z}\) and \(\mathbb{R}\) are not Eberlein groups one obtains actions of \(\mathbb{Z}\) and \(\mathbb{R}\) which are reflexively representable but not Hilbert representable. Moreover, it is proved that there exists a monothetic compact metric semitopological semigroup \(S\) which is reflexively representable but not Hilbert representable.

Reviewer: Peter Raith (Wien)

MSC:

37B99 Topological dynamics
54H15 Transformation groups and semigroups (topological aspects)
43A65 Representations of groups, semigroups, etc. (aspects of abstract harmonic analysis)
54H20 Topological dynamics (MSC2010)

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