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Solutions for Neumann boundary value problems involving $p(x)$-Laplace operators. (English)

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If $N \in \mathbb{N}$ and $\Omega \subseteq \mathbb{R}^N$ is a bounded domain with smooth boundary, consider the problem

$$\begin{cases} -\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u) + |u|^{p(x)-2}u = \lambda f(x,u), & \text{in } \Omega, \\ |\nabla u|^{p(x)-2}\frac{\partial u}{\partial \nu} = \mu g(x,u), & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $p \in C(\overline{\Omega})$, $p > 1$ in $\overline{\Omega}$, $\lambda, \mu \in \mathbb{R}$, and $\lambda^2 + \mu^2 > 0$. Moreover, throughout the paper f and g denote Caratheodory functions with subcritical growth in a suitable sense, with respect to the variable exponent $p(x)$. The author proves existence results that are analogues of classical results for the case $p \equiv 2$, using direct and minimax methods in a variational setting. The analogues for the following settings are covered: Sublinear and superlinear nonlinearities, superlinear odd nonlinearities, and odd concave-convex nonlinearities. Also the existence of a positive solution is considered in some of these cases.

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MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
35J60 Nonlinear elliptic equations
35J70 Degenerate elliptic equations
35J20 Variational methods for second-order elliptic equations

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