Dancer, E. N.

Finite Morse index solutions of supercritical problems. (English) Zbl 1158.35013

J. Reine Angew. Math. 620, 213-233 (2008).

The author studies properties of bounded solutions to

$$-\Delta u = f(u) \qquad \text{on } \mathbb{R}^N$$

that have finite Morse index, in the sense that the spectrum of $-\Delta - f'(u)$ in $L^2(\mathbb{R}^N)$ has only a finite number of negative points, each with finite algebraic multiplicity. Apparently, only $N \geq 2$ is considered. Denote $p^* := \infty$ if $N = 2$, $p^* := (N+2)/(N-2)$ if $N \geq 3$, $p^f := \infty$ if $N \leq 4$, and $p^f := N/(N-4)$ if $N > 4$. The main interest lies in the case where f grows supercritically at ∞, i. e., $|f(t)|/|t|^{p^*} \to \infty$ as $|t| \to \infty$.

The first result states that for $p^* < p \leq p^f$ and $f(t) := |t|^{p-1}t$ the only bounded finite Morse index solution is $u \equiv 0$. Similar results are proved for equations with homogeneous Dirichlet boundary conditions on a half space or in exterior domains. In some cases also $p = p^*$ is allowed.

In the second result the existence of a bounded, non-constant finite Morse index solution u is assumed, and conclusions are drawn concerning the shape of u and properties of f. More specifically, it is assumed that $f \in C^1$, $f \geq 0$ on $[\inf u, \sup u]$, and that f behaves like a power function near its zeros. Then u is radially symmetric with respect to some point and asymptotically constant at ∞. Moreover, some restrictions on the interaction of the zeros of f and the values of u are proved.

As one application the author proves a Bahri-Lions type result for supercritical problems: Boundedness of solutions is equivalent to boundedness of Morse indices, for a Dirichlet problem on a bounded domain. Other applications are the existence of infinitely many bifurcation points on the positive solution branch for a real analytic Dirichlet problem on a bounded, star shaped domain, and some results about domain variation in Dirichlet problems.

Reviewer: Nils Ackermann (México)

MSC:

35B35 Stability in context of PDEs
35B40 Asymptotic behavior of solutions to PDEs
35B45 A priori estimates in context of PDEs
35B25 Singular perturbations in context of PDEs
35J60 Nonlinear elliptic equations
47J30 Variational methods involving nonlinear operators
35J65 Nonlinear boundary value problems for linear elliptic equations

Keywords:

Morse index; bounded solution; supercritical nonlinearity

Full Text: DOI

References:

[1] DOI: 10.1090/S0894-0347-00-00345-3 · Zbl 0968.35041 · doi:10.1090/S0894-0347-00-00345-3
[2] DOI: 10.1002/cpa.3160450908 · Zbl 0801.35026 · doi:10.1002/cpa.3160450908
[3] Berestycki H., Ann. Scu. Norm. Sup. Pisa 25 pp 69– (1996)
[4] DOI: 10.1080/03605309708821315 · Zbl 0910.35048 · doi:10.1080/03605309708821315
[5] DOI: 10.1007/BF01243922 · Zbl 0755.35036 · doi:10.1007/BF01243922
[6] DOI: 10.1007/s002050000087 · Zbl 0962.76012 · doi:10.1007/s002050000087
[7] DOI: 10.1002/cpa.3160420304 · Zbl 0702.35085 · doi:10.1002/cpa.3160420304

[8] Clement P., Ann. Scu. Norm. Sup. Pisa 14 pp 97– (1987)
[9] DOI: 10.1090/S0002-9947-04-03543-3 · Zbl 1145.35369 · doi:10.1090/S0002-9947-04-03543-3
[10] DOI: 10.1512/iumj.2004.53.2354 · Zbl 1183.35125 · doi:10.1512/iumj.2004.53.2354
[11] Dancer E. N., Di{\currency}. Int. Eqns. 17 pp 961– (2004)
[12] DOI: 10.1007/s00208-002-0352-2 · Zbl 1040.35033 · doi:10.1007/s00208-002-0352-2
[13] DOI: 10.1016/0022-0396(88)90021-6 · Zbl 0662.34025 · doi:10.1016/0022-0396(88)90021-6
[14] Dancer E. N., Topol. Meth. Nonlin. Anal. 7 pp 95– (1996)
[15] DOI: 10.1007/BF02505896 · Zbl 1030.35073 · doi:10.1007/BF02505896
[16] DOI: 10.1007/PL00004666 · Zbl 0933.35068 · doi:10.1007/PL00004666
[17] Dancer E. N., Di{\currency}. Int. Eqns. 5 pp 903– (1992)
[18] DOI: 10.1017/S0004972700012089 · Zbl 0777.35005 · doi:10.1017/S0004972700012089
[19] DOI: 10.1016/S0362-546X(98)00307-1 · Zbl 0960.35035 · doi:10.1016/S0362-546X(98)00307-1
[20] DOI: 10.1007/BF01209398 · Zbl 0608.35017 · doi:10.1007/BF01209398
[21] Farina A., CR Acad. Sci. Paris 341 pp 415– (2005)
[22] DOI: 10.1002/cpa.3160340406 · Zbl 0465.35003 · doi:10.1002/cpa.3160340406
[23] Joseph D., Arch. Rat. Mech. Anal. 49 pp 241– (1973)
[24] Moschini L., Anal. Nonlin. 22 pp 11– (2005)
[25] DOI: 10.1002/cpa.3160380105 · Zbl 0581.35021 · doi:10.1002/cpa.3160380105
[26] DOI: 10.1006/jfan.1993.1064 · Zbl 0793.35039 · doi:10.1006/jfan.1993.1064
[27] DOI: 10.1080/03605307908820096 · Zbl 0462.35016 · doi:10.1080/03605307908820096
[28] Schaaf R., Adv. Di{\currency}. Eqns. 5 pp 1201– (2000)
[29] DOI: 10.1006/jdeq.1995.1105 · Zbl 0844.35028 · doi:10.1006/jdeq.1995.1105
[30] DOI: 10.1512/iumj.1996.45.986 · Zbl 0864.35009 · doi:10.1512/iumj.1996.45.986